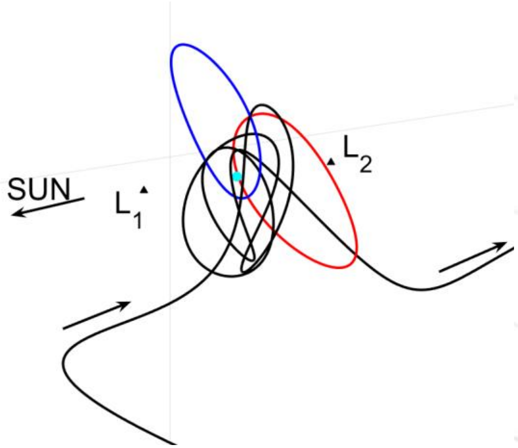

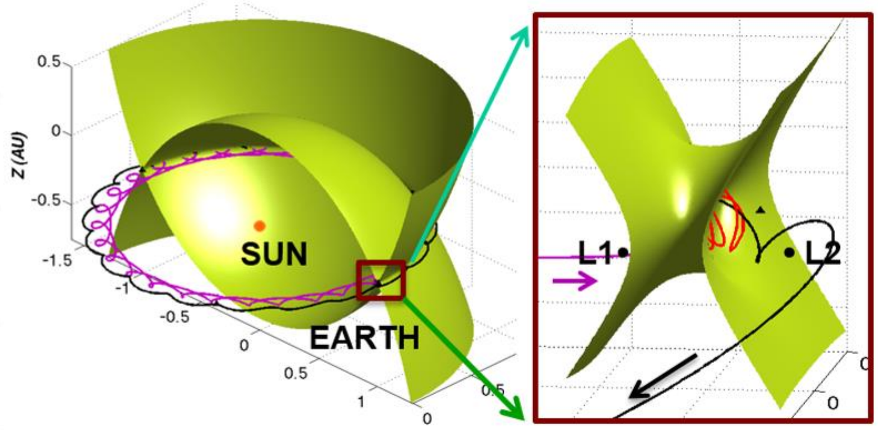

DYNAMICS OF ASTEROID 2006 RH120: TEMPORARY CAPTURE PHASE

28TH AAS/AIAA SPACE FLIGHT MECHANICS MEETING, KISSIMMEE, FL

BRIAN D. ANDERSON, UNIVERSITY OF SOUTHERN CALIFORNIA

JET PROPULSION LABORATORY, CALIFORNIA INSTITUTE OF TECHNOLOGY

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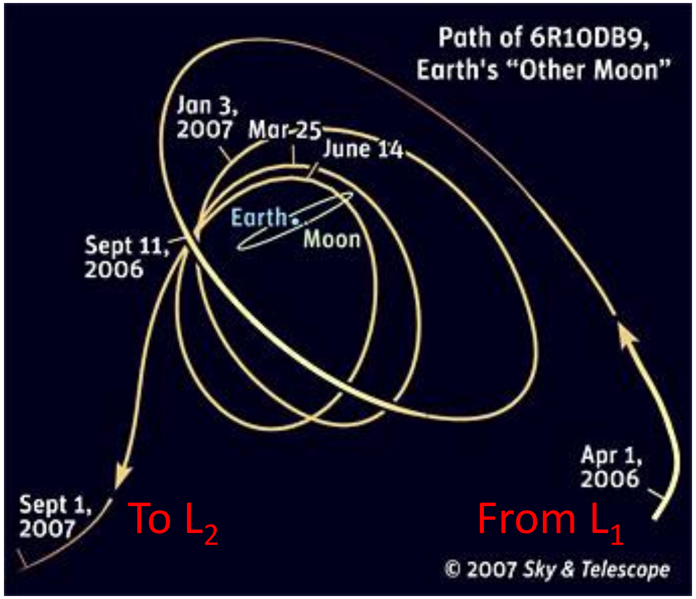
THURSDAY, JANUARY 11, 2018

Asteroid 2006 RH120 Temporary Capture

- **Discovered 14 September 2006**
- **~5 m Diameter**
- **Orbited Earth 3 times (~1 year)**
- **Closest perigee ~0.7 Lunar Distances**
- **Minimoon: Granvik, Vaubaillon and Jedicke 2012 (may be abundant!)**
- **Prime target for low cost rendezvous, retrieval missions.**
- **Pre & Post Capture Phase (2016 Work): Dynamics controlled by invariant manifolds of resonant orbits and L_1 & L_2 halo orbits.**

Sky & Telescope



- **Capture Phase Surprising Result (Current Work):**

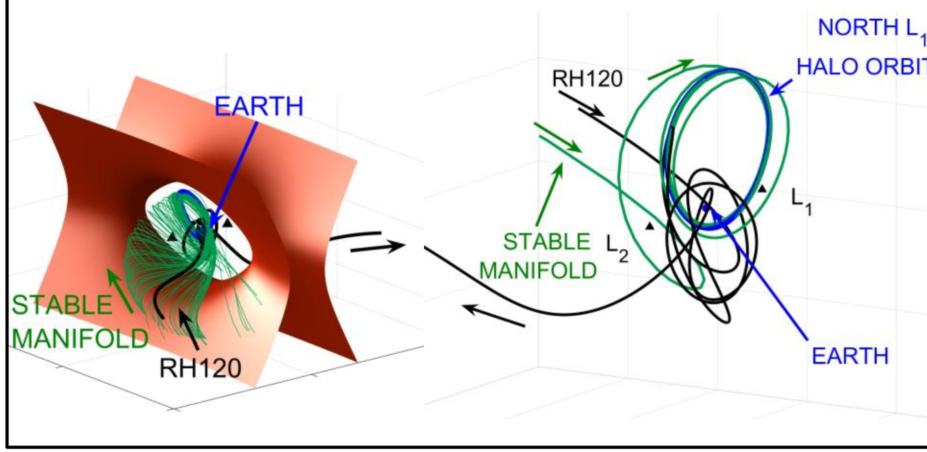
  Eccentricity of Earth orbit dominates the capture dynamics

  - **Elliptic Restricted 3 Body Model provides better overall performance than restricted 4 body models.**
- **Developed new metric for comparison of orbits in different dynamical models: Modified Dynamic Time Warping**
  - **Derived from shape analysis of signal processing**

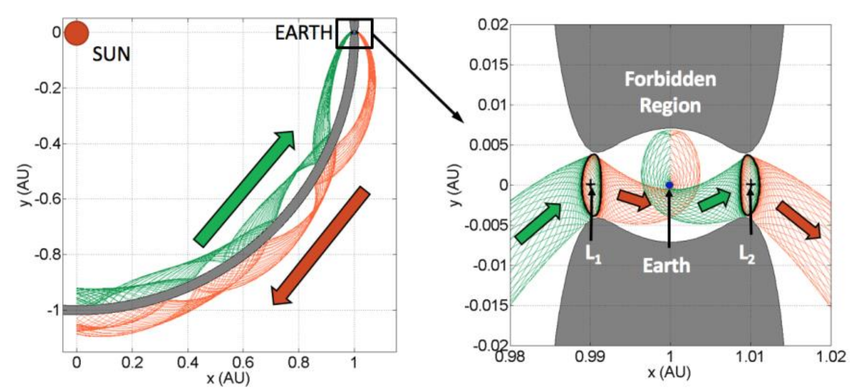
Pre & Post Capture by Invariant Manifolds (2016)

- Temporary capture of Asteroid 2006 RH120 enabled by repeated resonant close encounters with Earth
- Invariant manifolds control asteroid capture
  - Captured via **stable manifold** of L_1 North Halo Orbit
  - Escaped via **unstable manifold** of L_2 South Halo Orbit

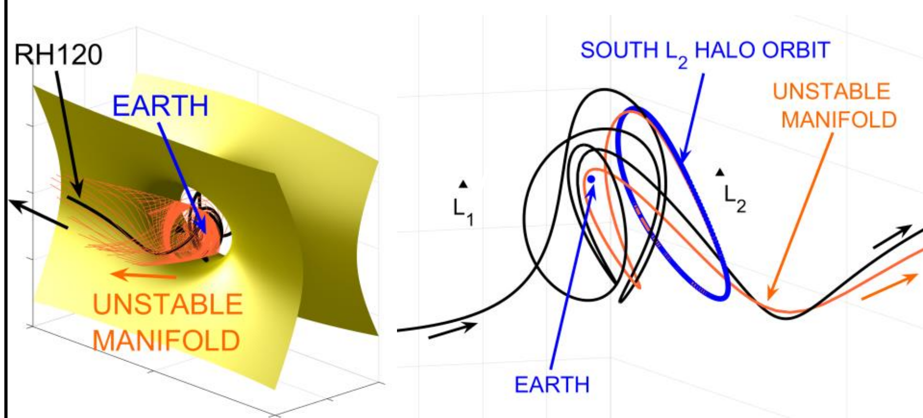
ASTEROID CAPTURE DYNAMICS



MANIFOLD TRANSIT DYNAMICS

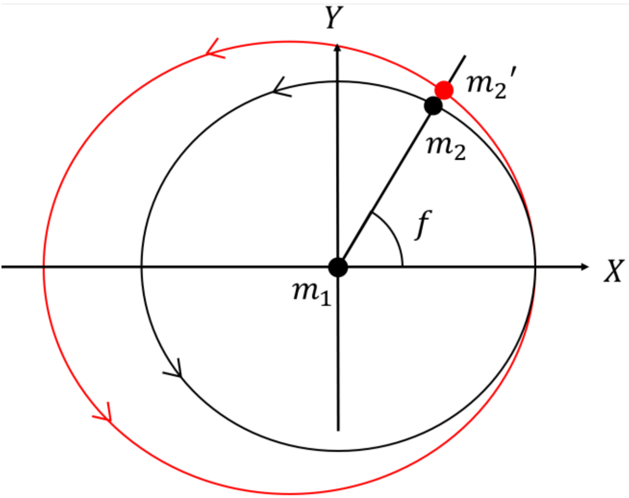


ASTEROID ESCAPE DYNAMICS

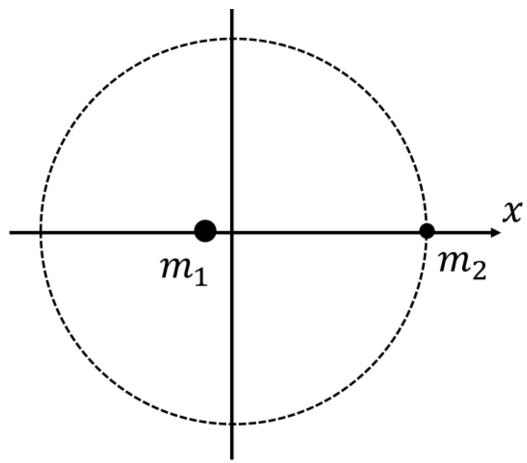


3 Dynamical Systems Models

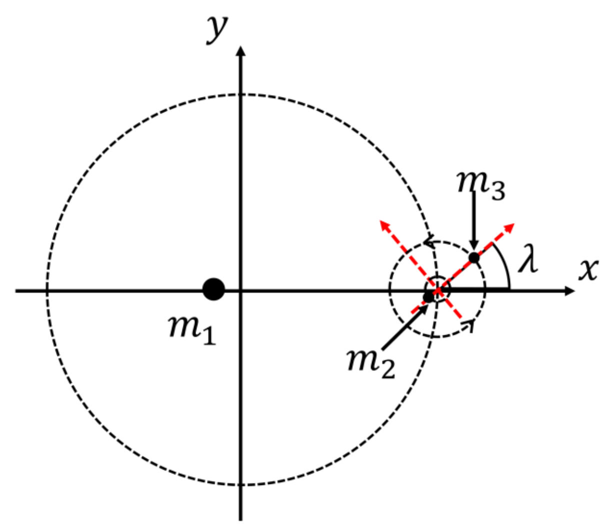
Inertial CR3BP & ER3BP



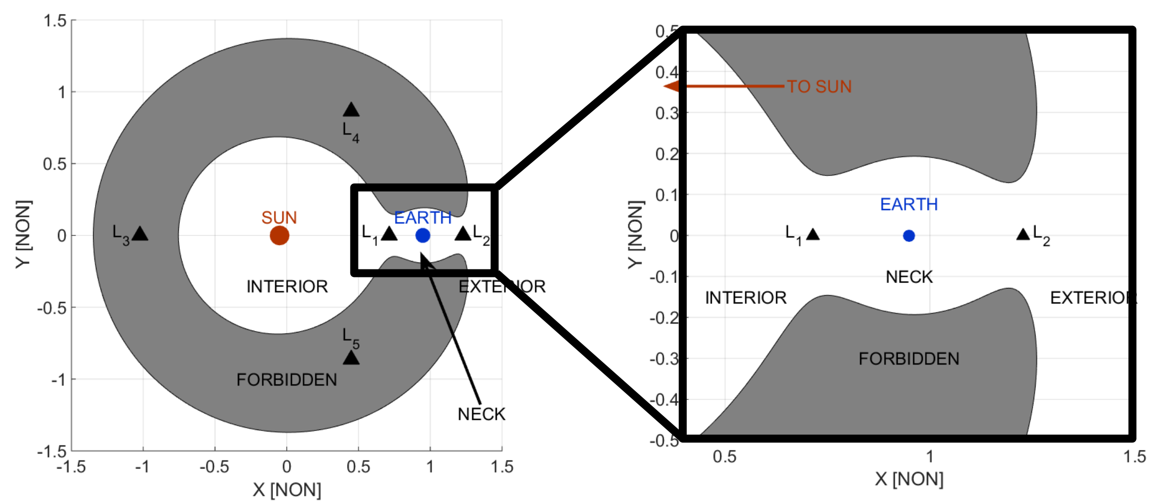
***Rotating* CR3BP & *Pulsating* ER3BP**



***Rotating* BCP**

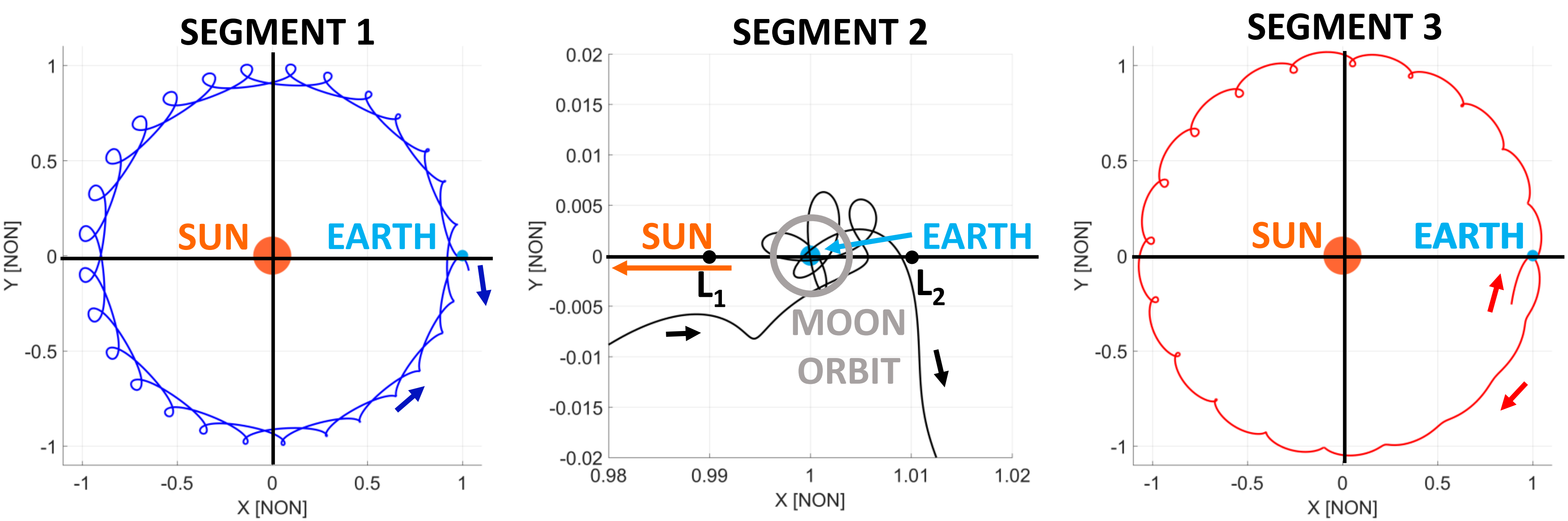


- CR3BP and ER3BP rotating frames look identical
  - Pulsating coordinates keep m_1 and m_2 fixed on ER3BP *x*-axis
- BCP places m_2-m_3 barycenter where m_2 is in CR3BP frame
- CR3BP has Jacobi Integral
  - Forbidden Region fixed
- BCP & ER3BP no Integral
  - Forbidden Region variable



Segment Definitions & Approach

- Show which dynamical systems provides best model for each orbit phase, best model for coupling between different orbit phases.
  - Coupling of Precapture Phase with Capture Phase orbits: Segment 1
  - Capture Phase orbit by itself: Segment 2
  - Coupling of Postcapture Phase with Capture Phase orbits: Segment 3



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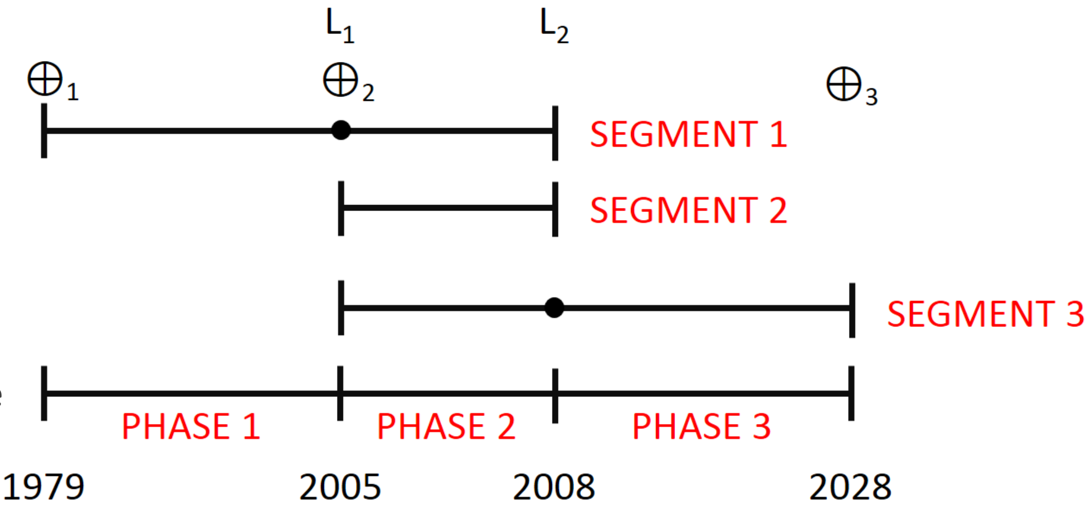
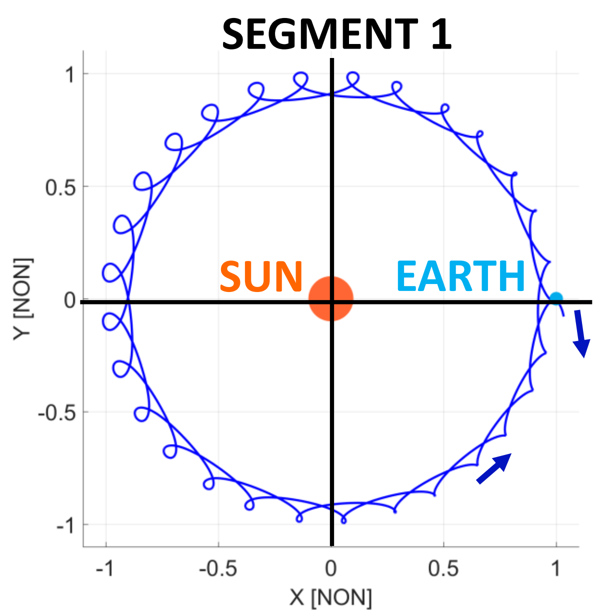
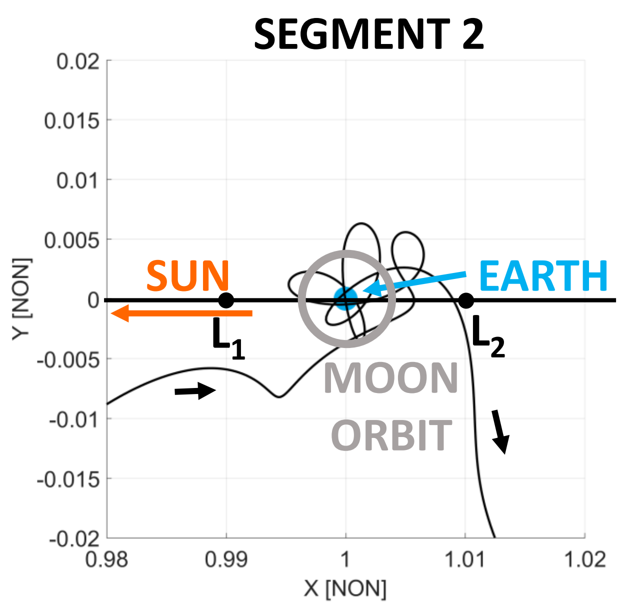
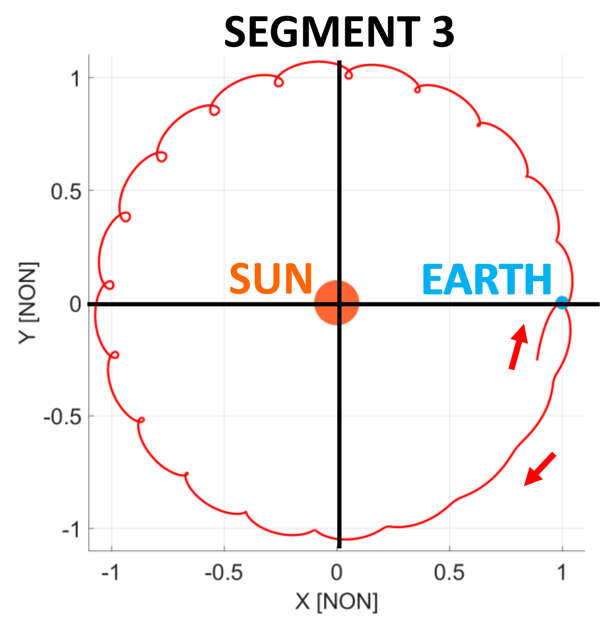
Segment Definitions

- Segment 1
  - Interior resonance + temporary capture
- Segment 2
  - Temporary capture
- Segment 3
  - Temporary capture + exterior resonance



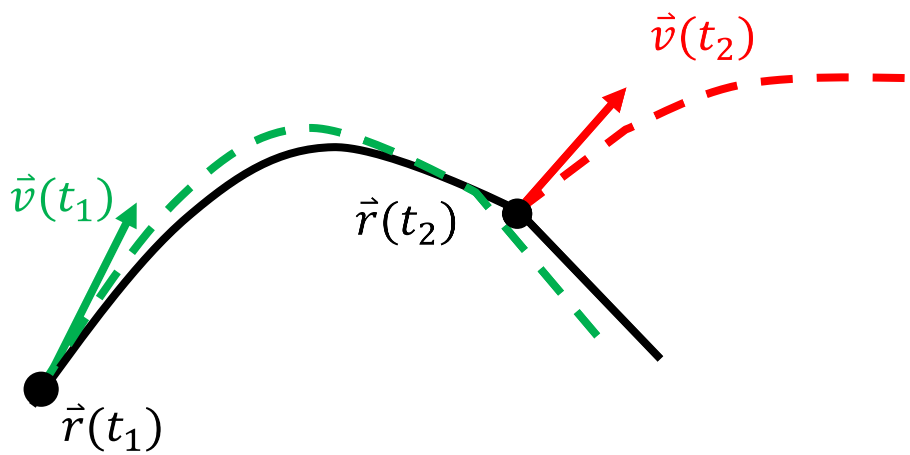






State Coherence Parameter Q

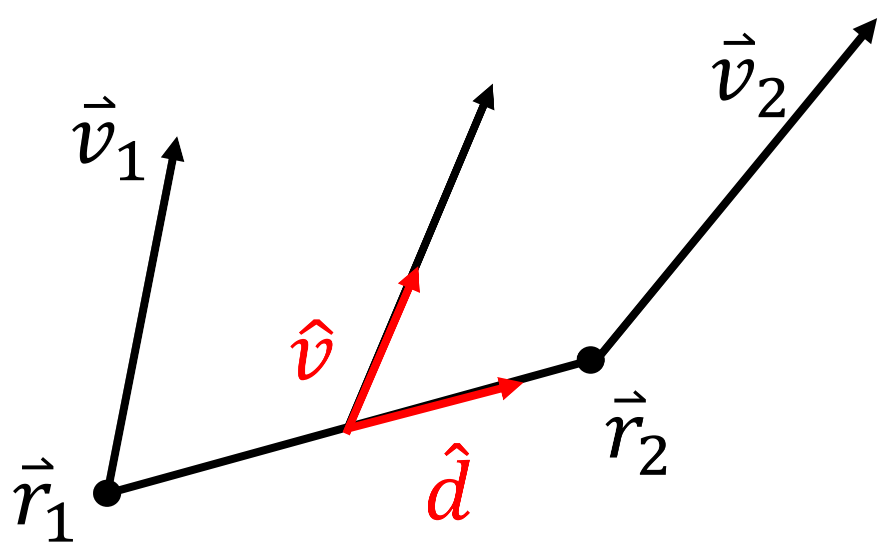
- Conversion of ephemeris to rotating frame may be incoherent
  - i.e. velocity not tangent to path
- Integration of incoherent state yields wrong results
  - **Red** curve is incoherent
  - **Green** curve is coherent



- Coherence Parameter $Q = \hat{\boldsymbol{v}} \cdot \hat{\boldsymbol{d}}$
  - $Q = 1$, fully coherent
  - $Q = 0$, fully incoherent

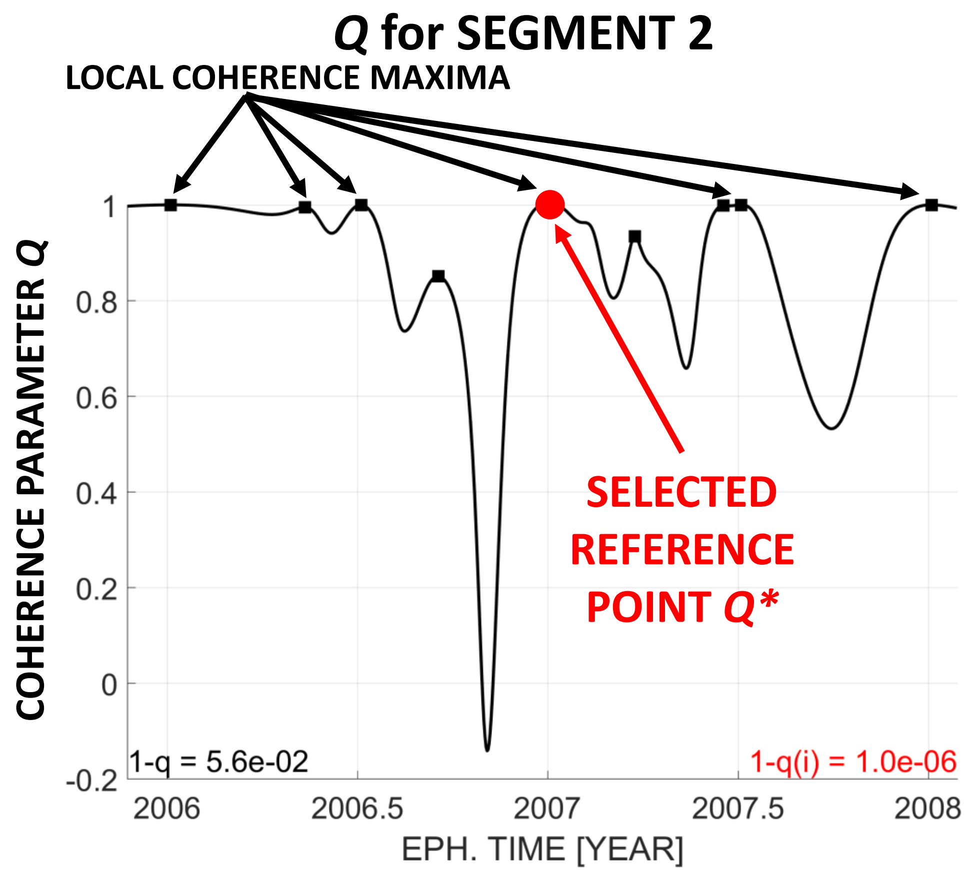
$$\hat{d} = \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|}$$

$$\hat{v} = \frac{\vec{v}_2 + \vec{v}_1}{|\vec{v}_2 + \vec{v}_1|}$$



Frame Conversion Coherence Curve

- Convert ephemeris state to rotating frame and units
- Compute coherence across trajectory
- Select "optimal" reference point *Q**
  from local coherence maxima
- Integrate in different models forward and/or backward in time using reference *Q**
- Compute similarity with DTW



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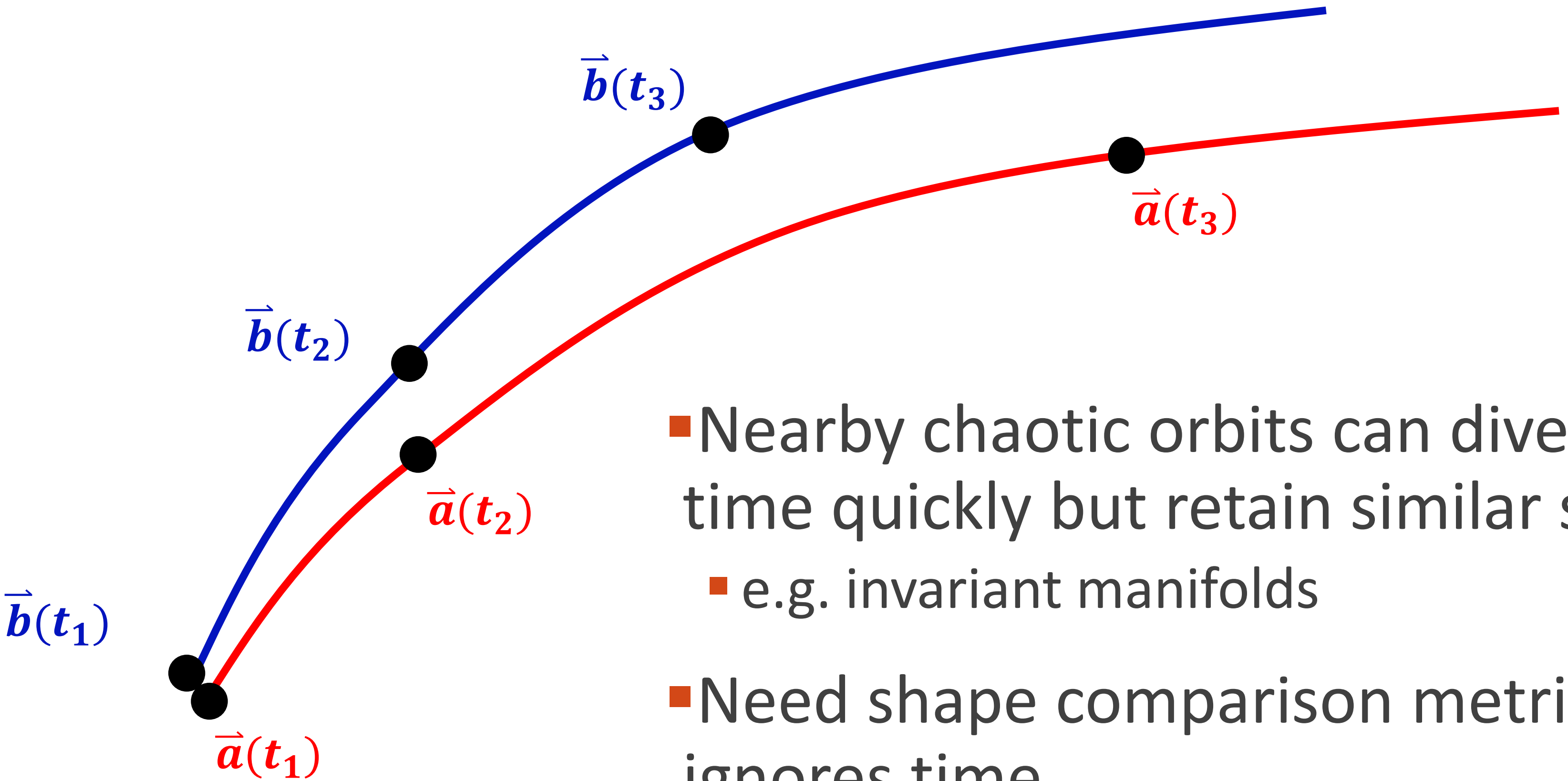
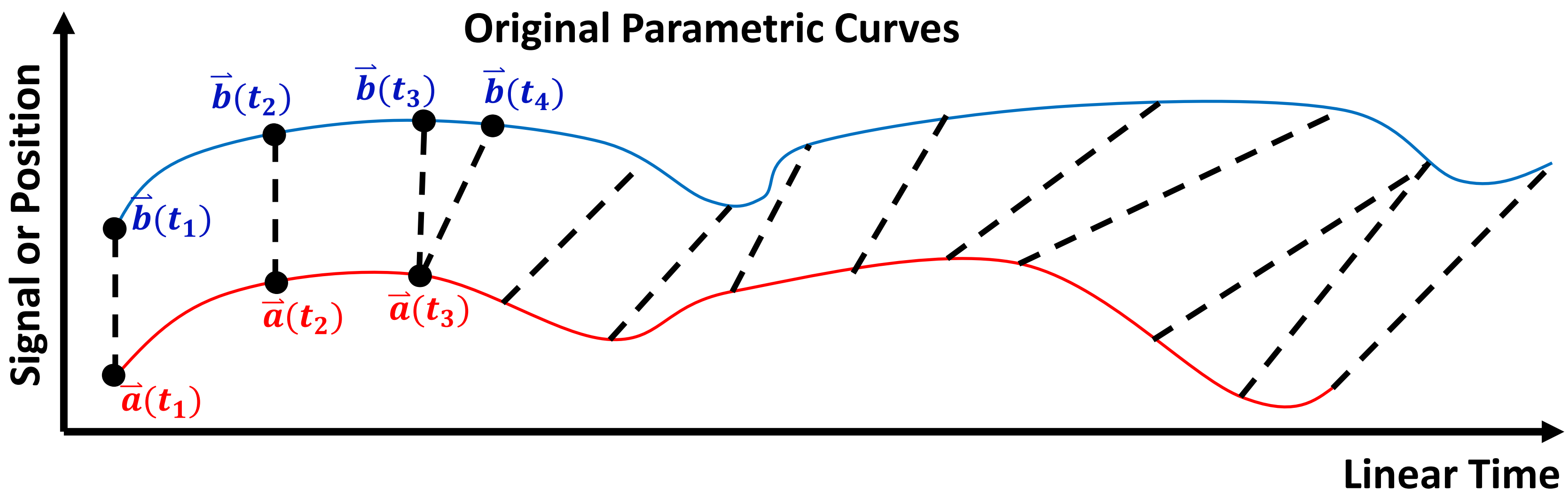
Comparing Chaotic Orbits



- Nearby chaotic orbits can diverge in time quickly but retain similar shape
  - e.g. invariant manifolds
- Need shape comparison metric that ignores time
  - **DYNAMIC TIME WARPING**

Dynamic Time Warping

- How similar is curve a to curve b?
- Shape only, ignore time stretching
- Locate sequence of pairs that minimizes cumulative distance between curves (for details see Sakoe & Chiba [1978])



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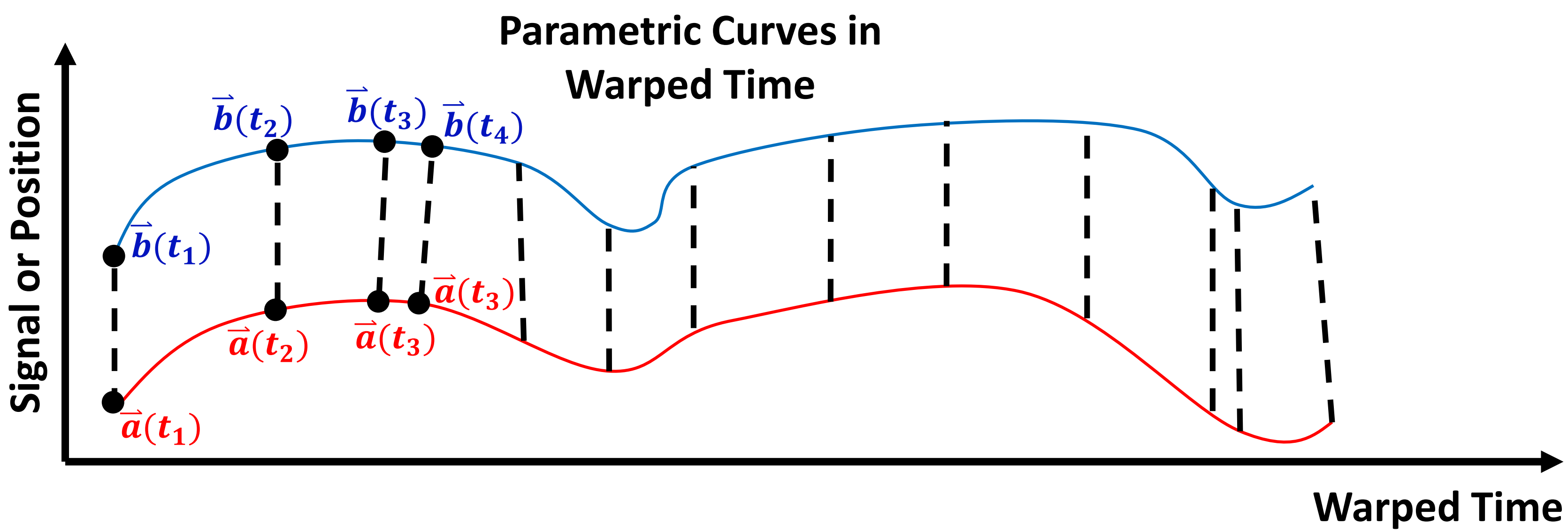
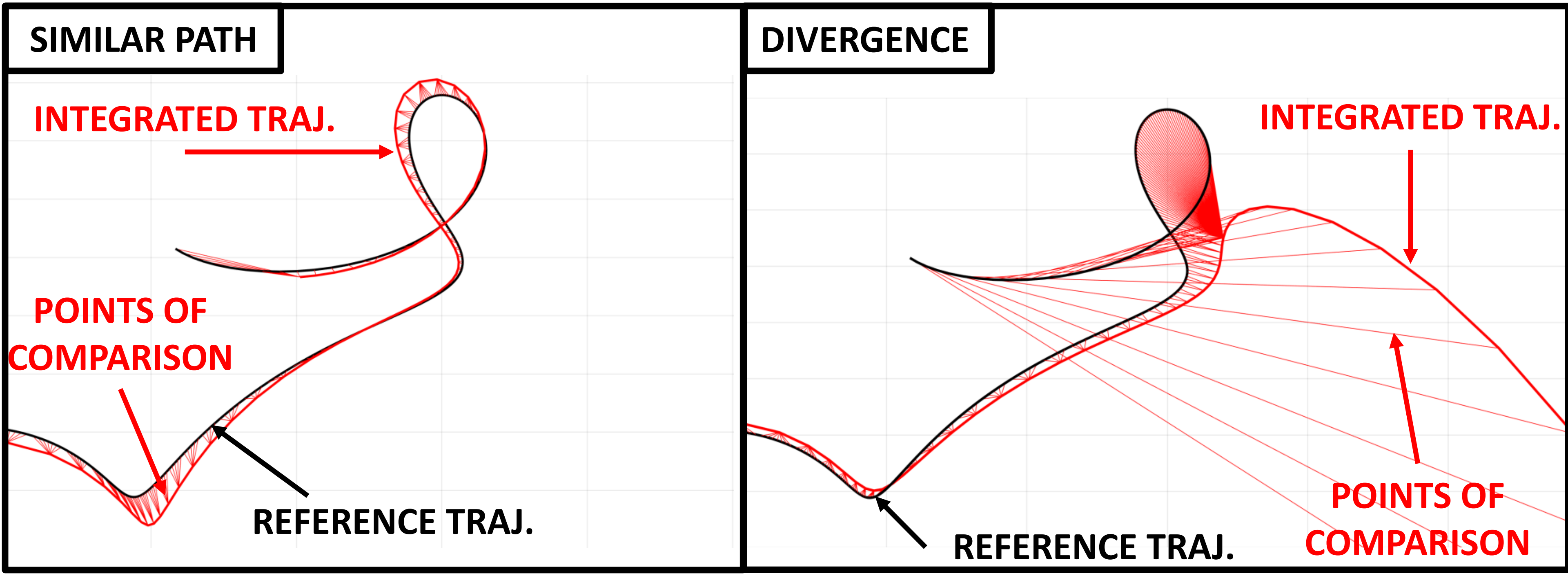
Dynamic Time Warping

- How similar is curve a to curve b?
- Shape only, ignore time stretching
- Locate sequence of pairs that minimizes cumulative distance between curves (for details see Sakoe & Chiba [1978])
- Note that a point on one curve can repeat



DTW Applied to Trajectories

- Lines show points of comparison between trajectories a and b
- When the path of two curves diverge significantly, points of comparison become extremely stretched



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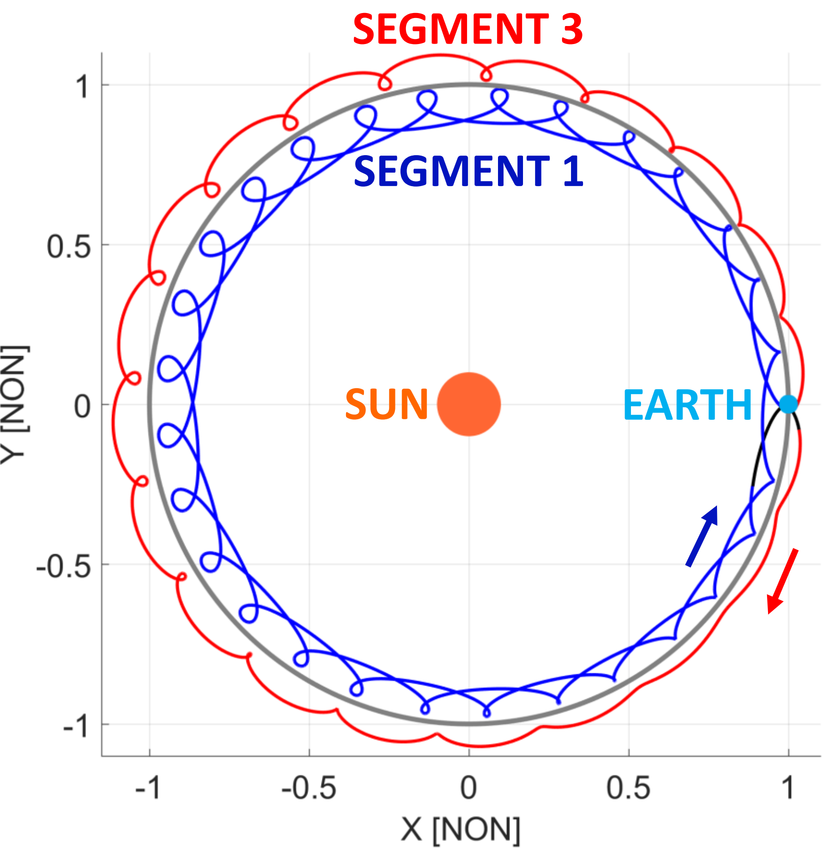
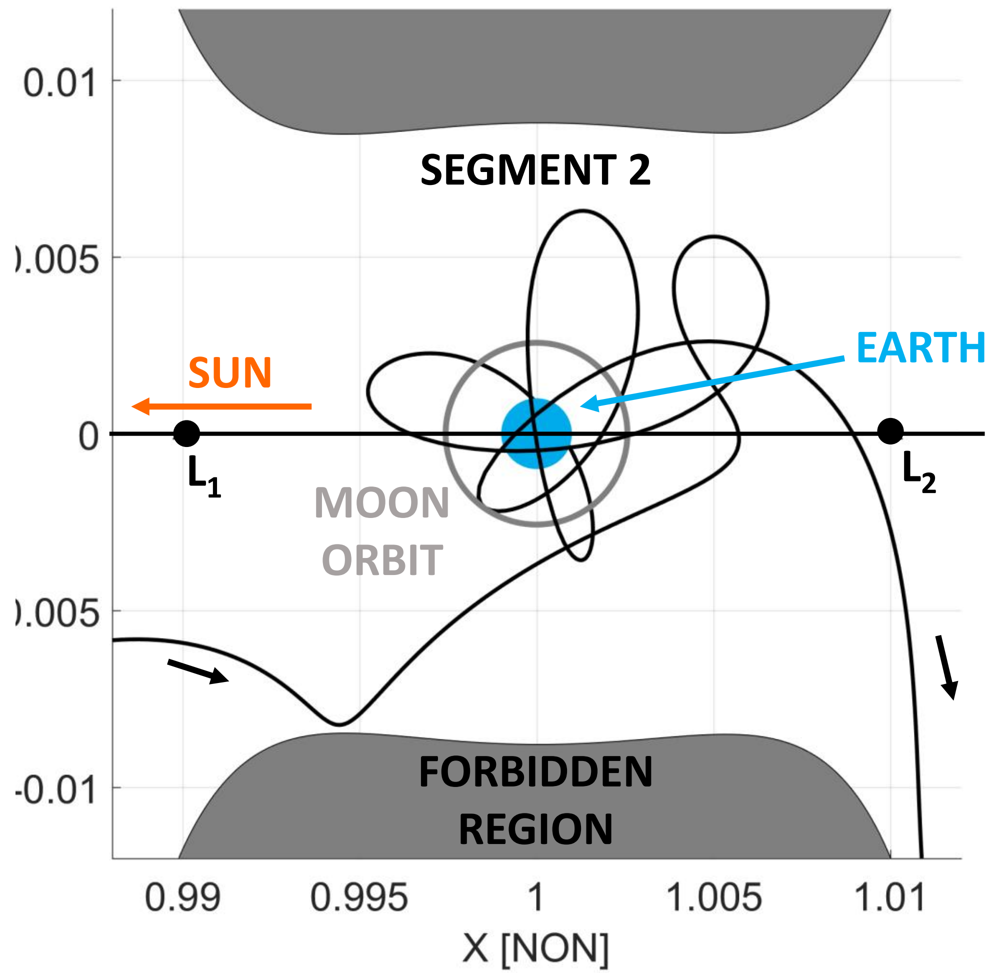
True Asteroid Orbit Geometry



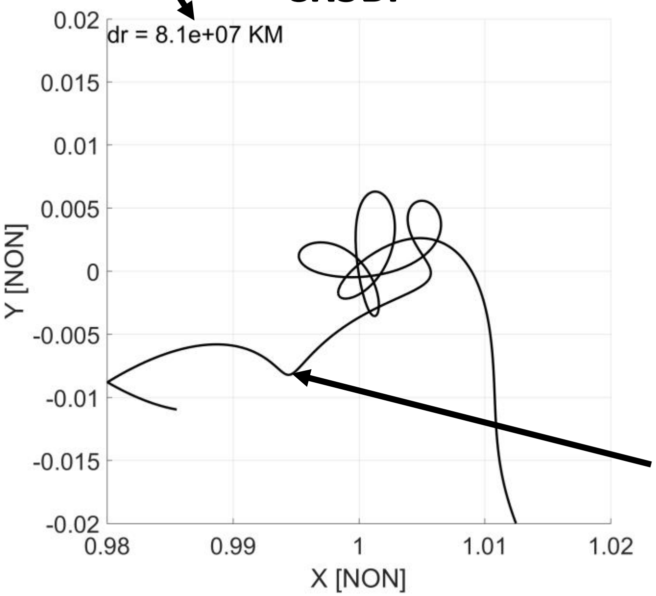


Results Geometry

- Graphical comparison results for segment 1
- First we show only the **asteroid trajectory** in rotating frame

Average DTW distance

CR3BP



Asteroid Trajectory

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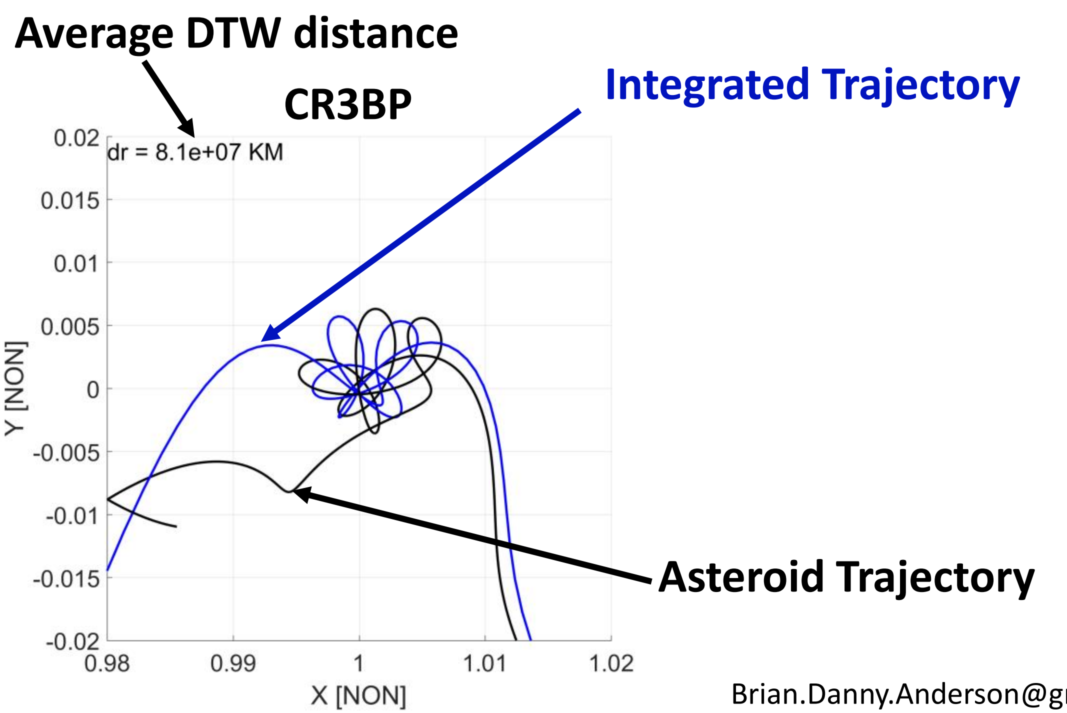
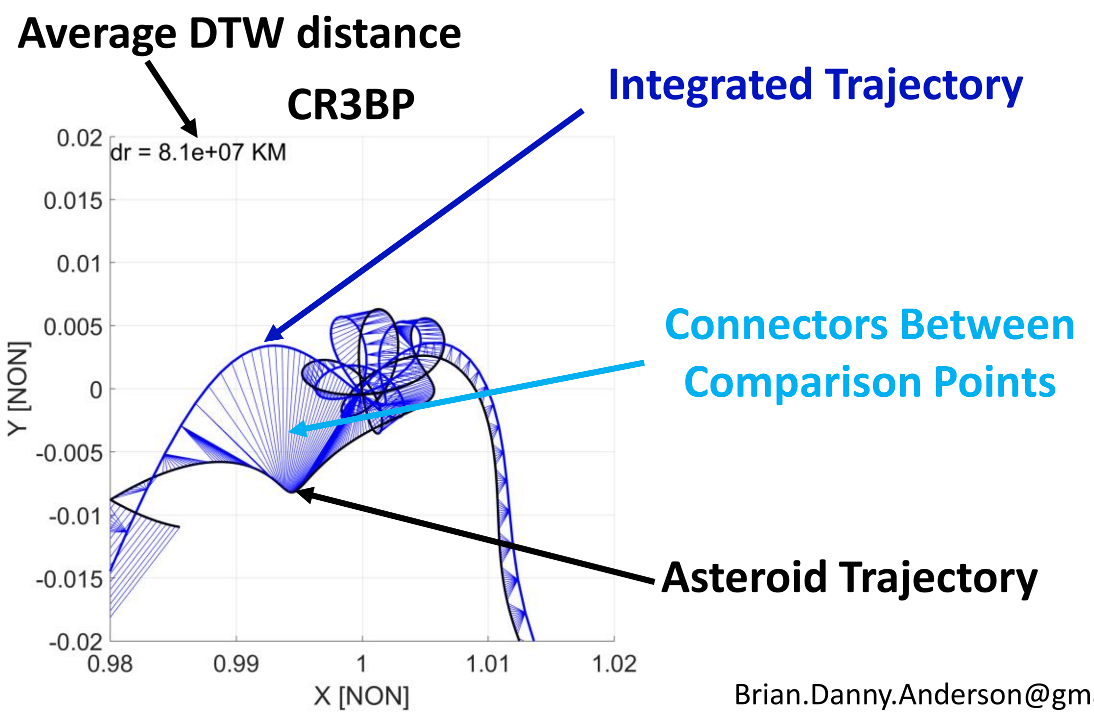
Results Geometry

- Graphical comparison results for segment 1
- First we show only the **asteroid trajectory** in rotating frame
- Then show the **integrated trajectory** in the CR3BP



Results Geometry

- Graphical comparison results for segment 1
- First we show only the **asteroid trajectory** in rotating frame
- Then show the **integrated trajectory** in the CR3BP
- Last show **points of comparison** used by the DTW algorithm as connectors

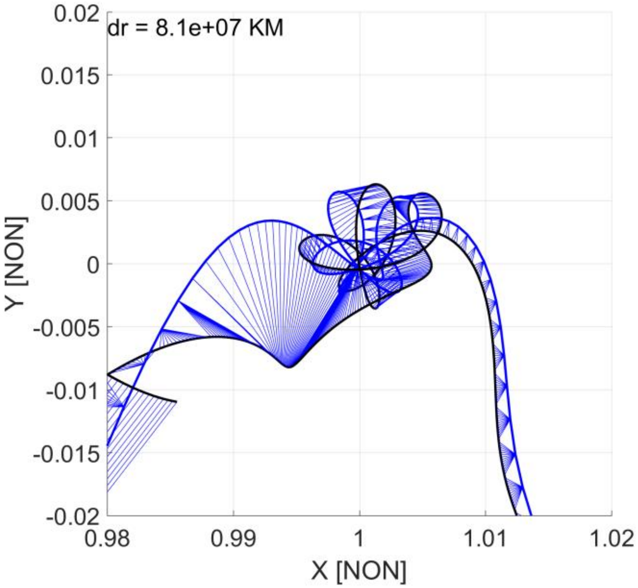


Results Geometry

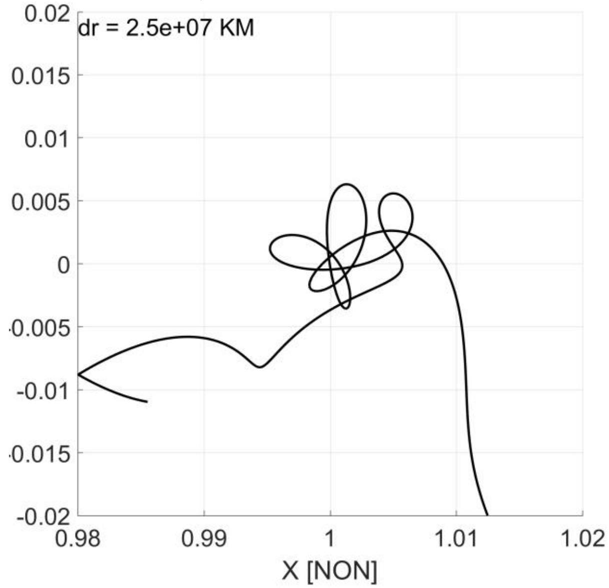
- Repeat procedure for BCP model results
- **Asteroid trajectory**

Average DTW distance

CR3BP



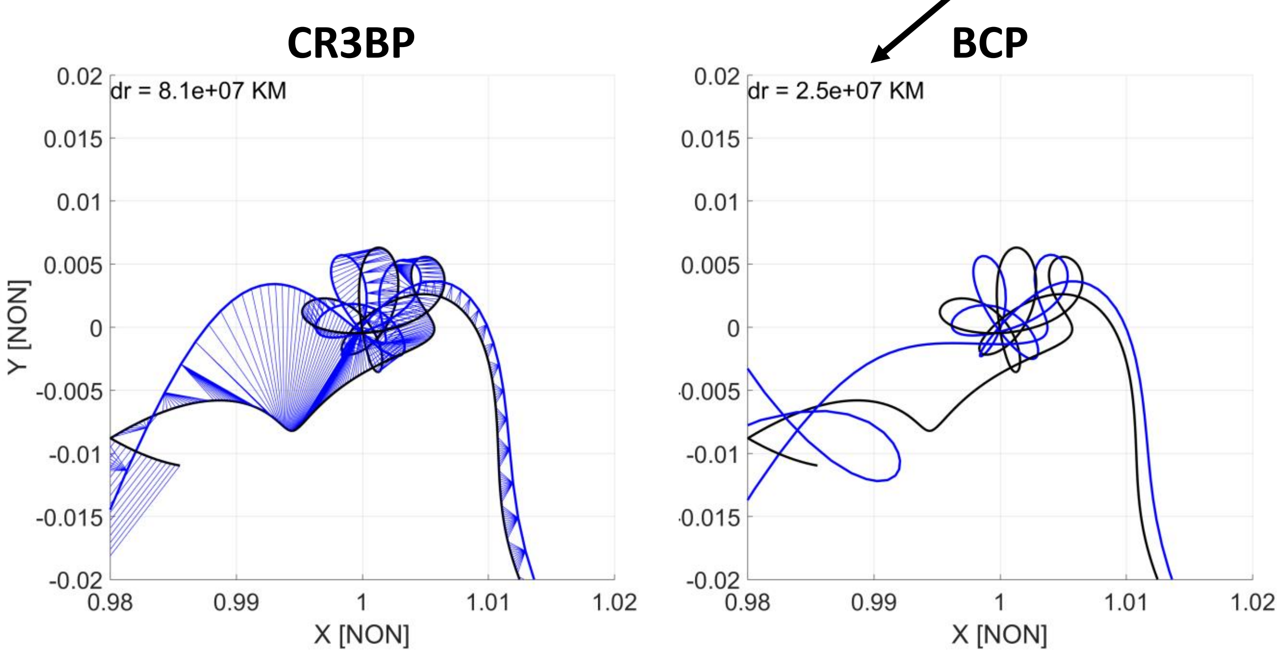
BCP



Results Geometry

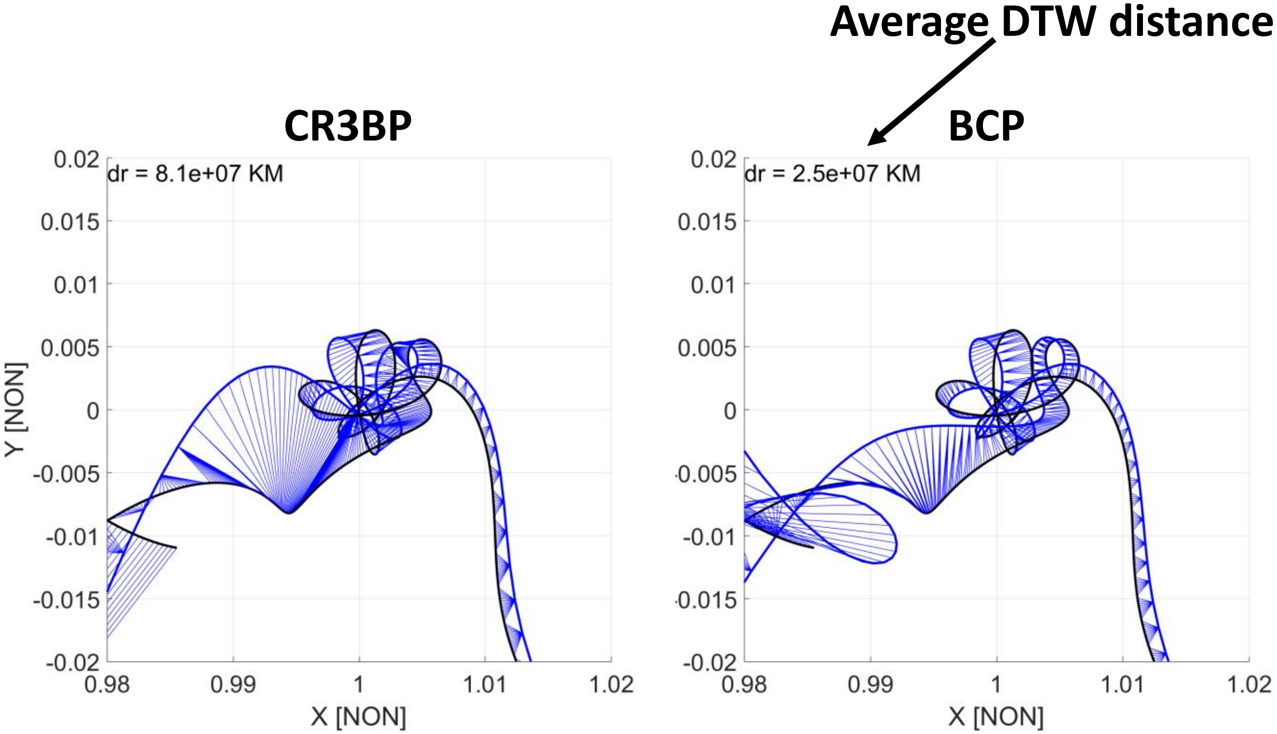
- Repeat procedure for BCP model results
- **Asteroid trajectory**
- **Integrated trajectory**

Average DTW distance



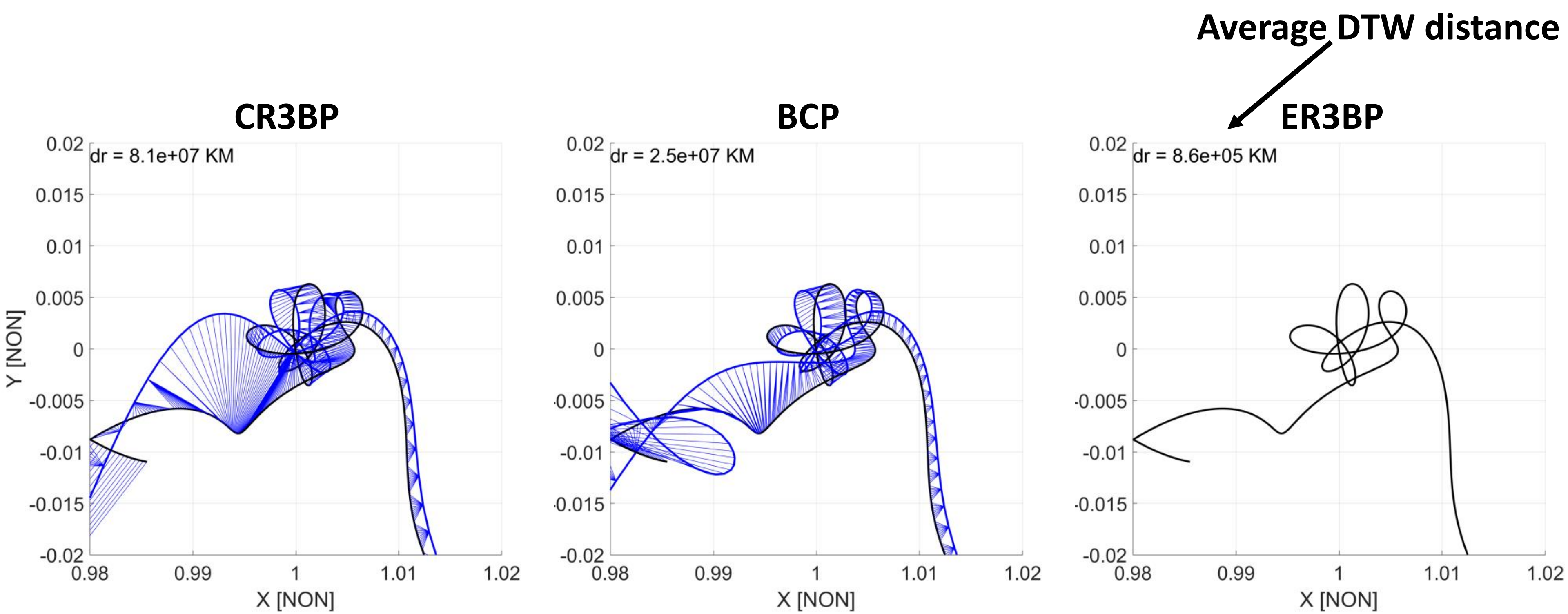
Results Geometry

- Repeat procedure for BCP model results
- **Asteroid trajectory**
- **Integrated trajectory**
- **Points of comparison**



Results Geometry

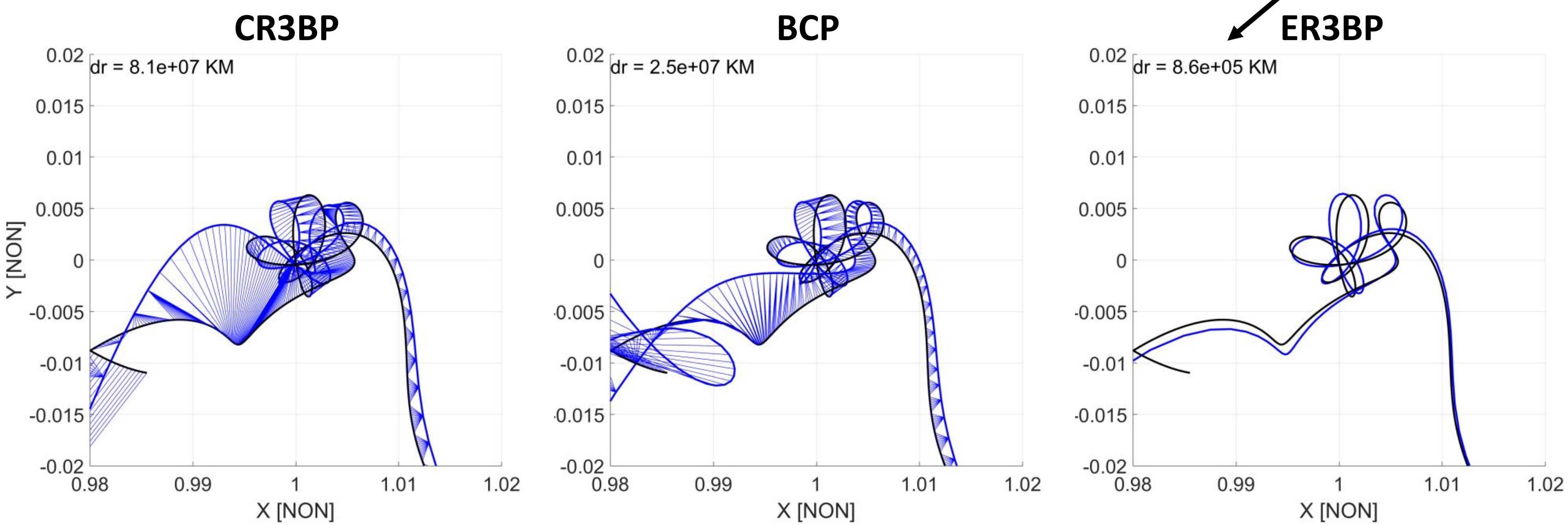
- Repeat procedure for ER3BP model results
- **Asteroid trajectory**



Results Geometry

- Repeat procedure for ER3BP model results
- **Asteroid trajectory**
- **Integrated trajectory**

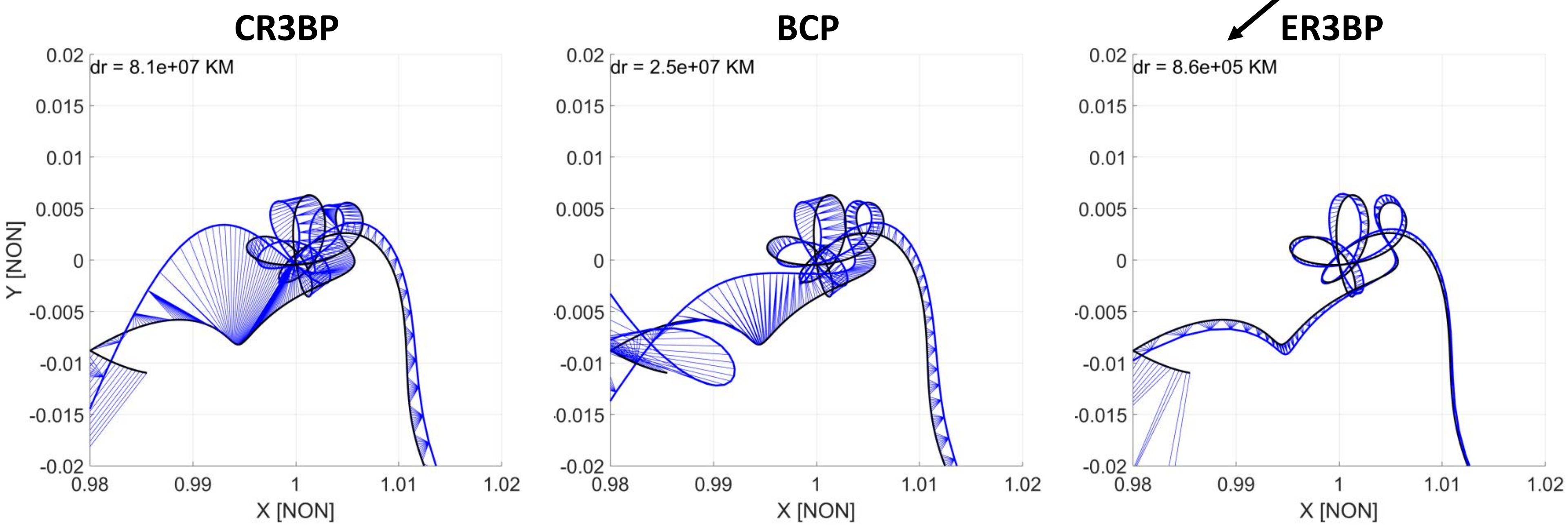
Average DTW distance



Results Geometry

- Repeat procedure for ER3BP model results
- **Asteroid trajectory**
- **Integrated trajectory**
- **Points of comparison**
- For this segment, the ER3BP matched the Asteroid Trajectory best.

Average DTW distance



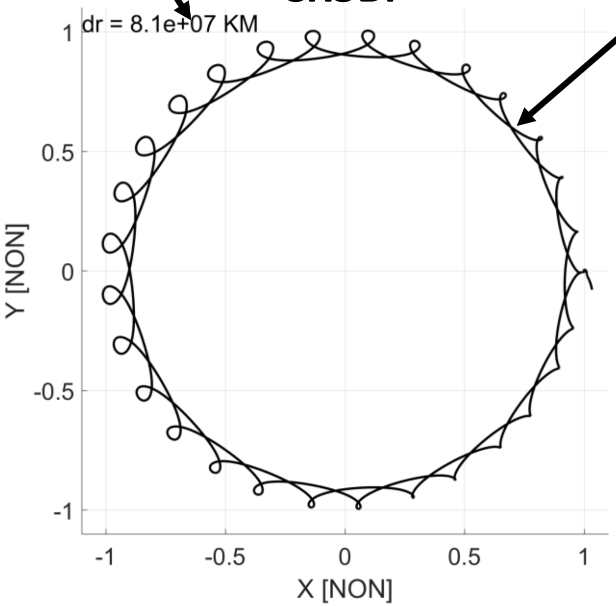
Results Geometry

- Graphical comparison results for segment 1
- First we show only the **asteroid trajectory** in rotating frame

Average DTW distance

Asteroid Trajectory

CR3BP



Results Geometry

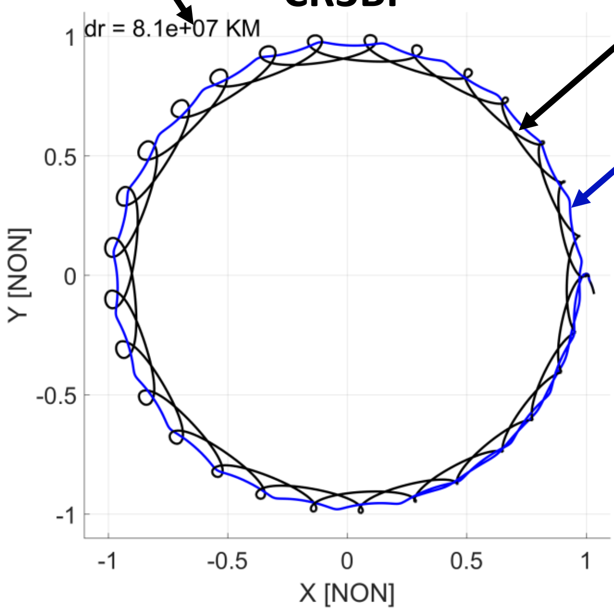
- Graphical comparison results for segment 1
- First we show only the **asteroid trajectory** in rotating frame
- Then show the **integrated trajectory** in the CR3BP

Average DTW distance

CR3BP

Asteroid Trajectory

Integrated Trajectory

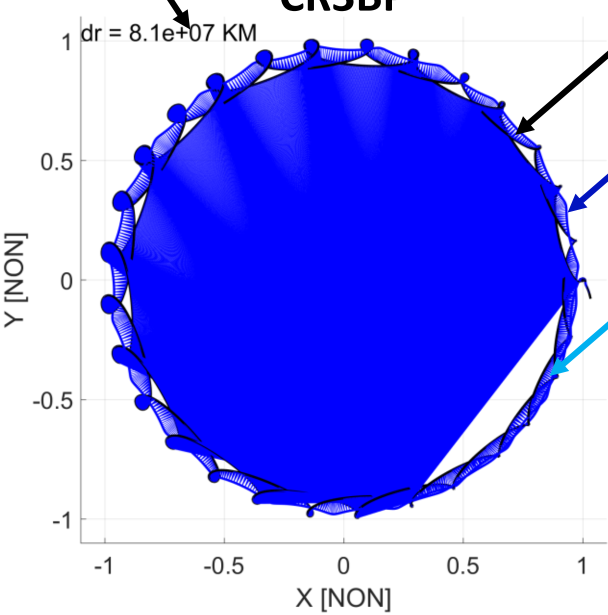


Results Geometry

- Graphical comparison results for segment 1
- First we show only the **asteroid trajectory** in rotating frame
- Then show the **integrated trajectory** in the CR3BP
- Last show **points of comparison** used by the DTW algorithm as connectors

Average DTW distance

CR3BP



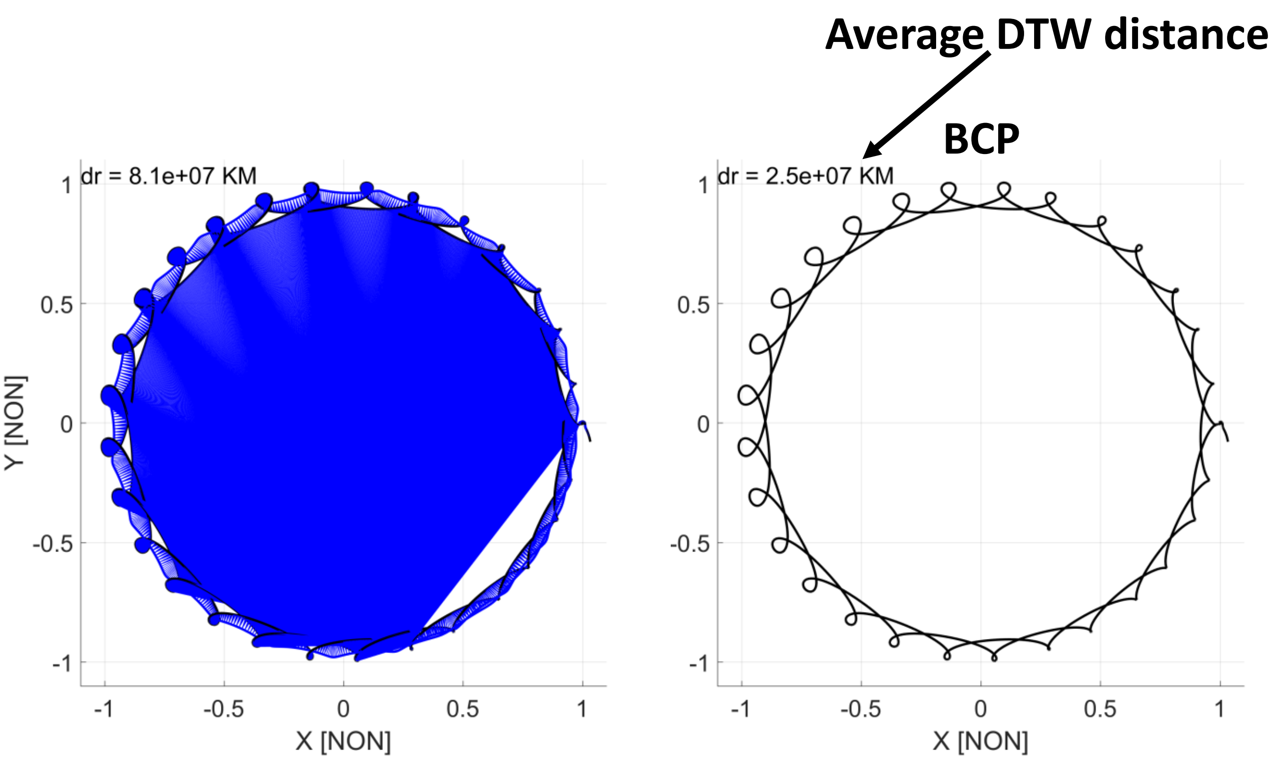
Asteroid Trajectory

Integrated Trajectory

Connectors Between Comparison Points

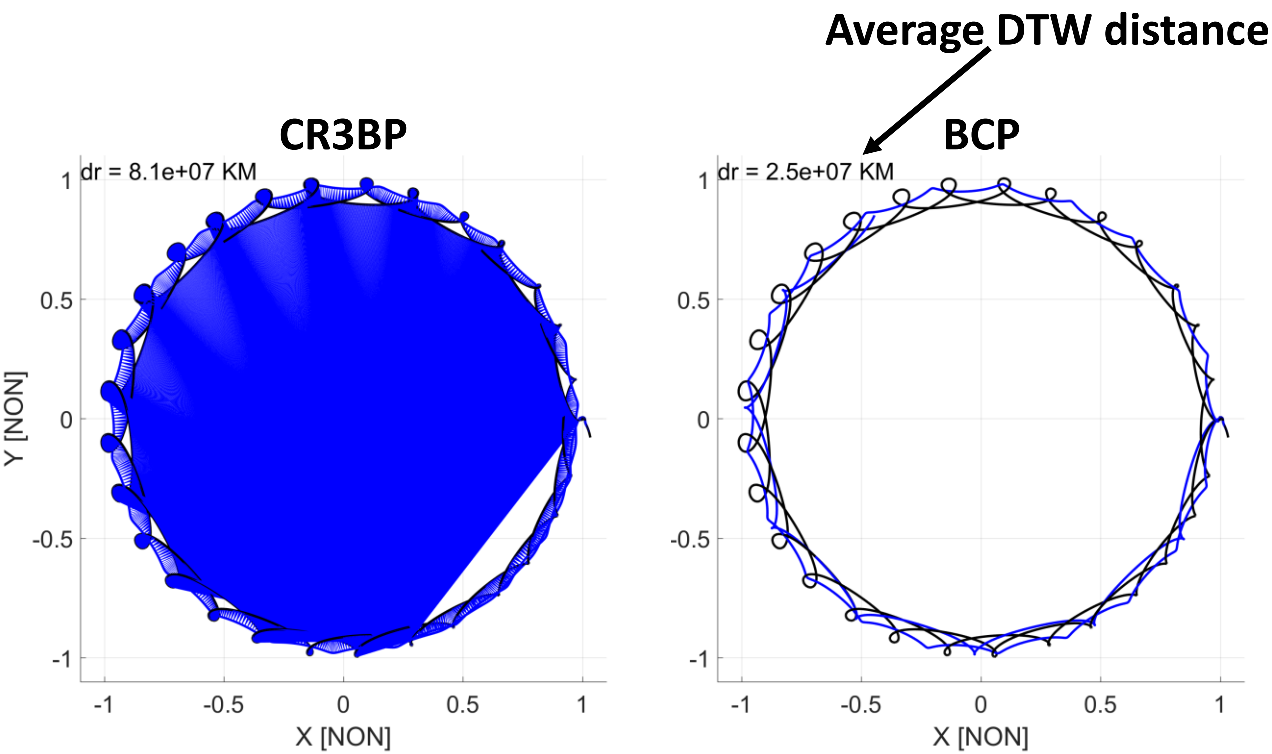
Results Geometry

- Repeat procedure for BCP model results
- **Asteroid trajectory**



Results Geometry

- Repeat procedure for BCP model results
- **Asteroid trajectory**
- **Integrated trajectory**

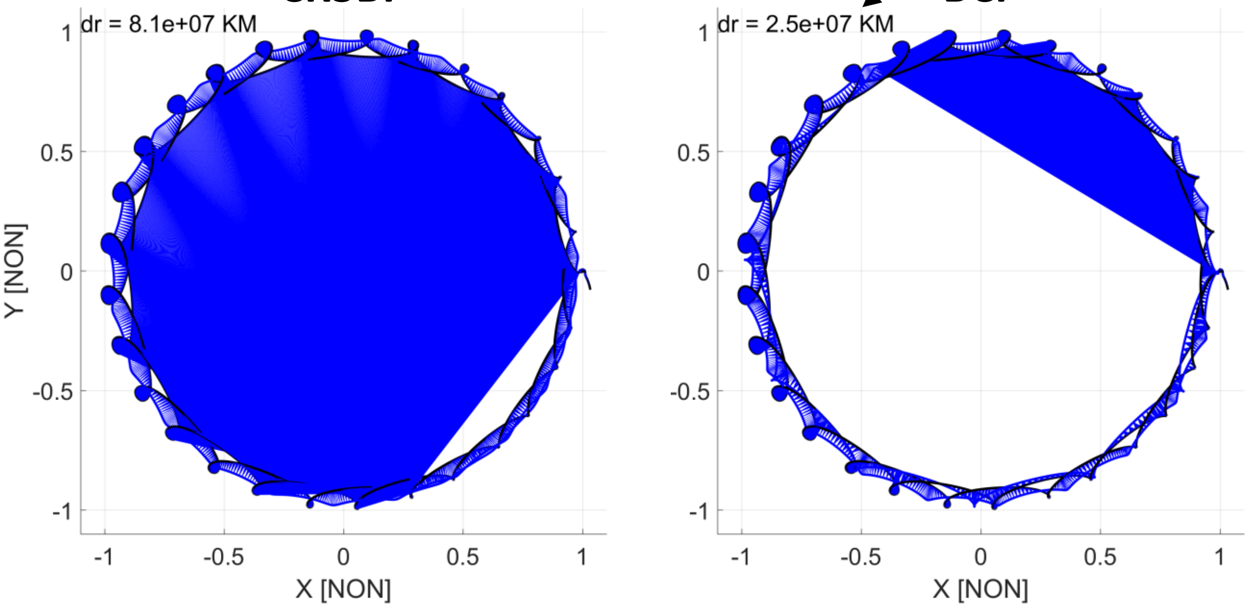


Results Geometry

- Repeat procedure for BCP model results
- **Asteroid trajectory**
- **Integrated trajectory**
- **Points of comparison**

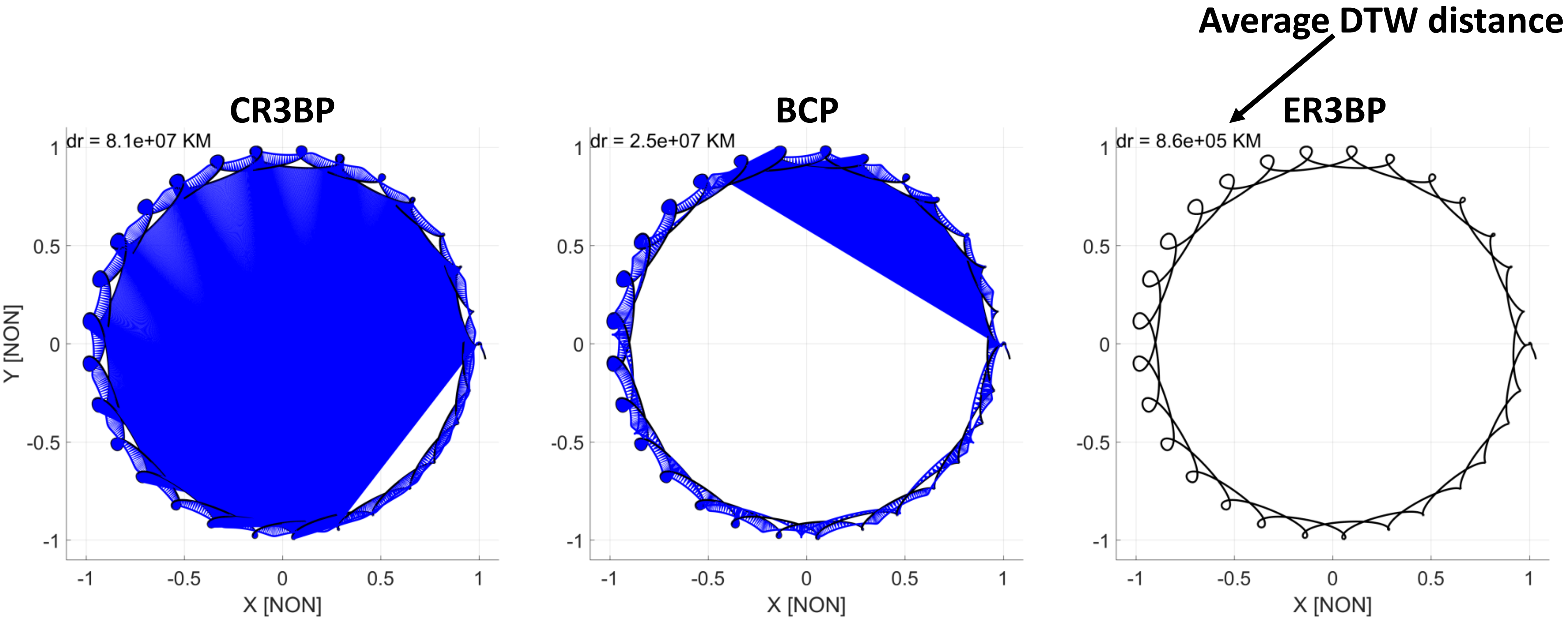
Average DTW distance

CR3BP **BCP**



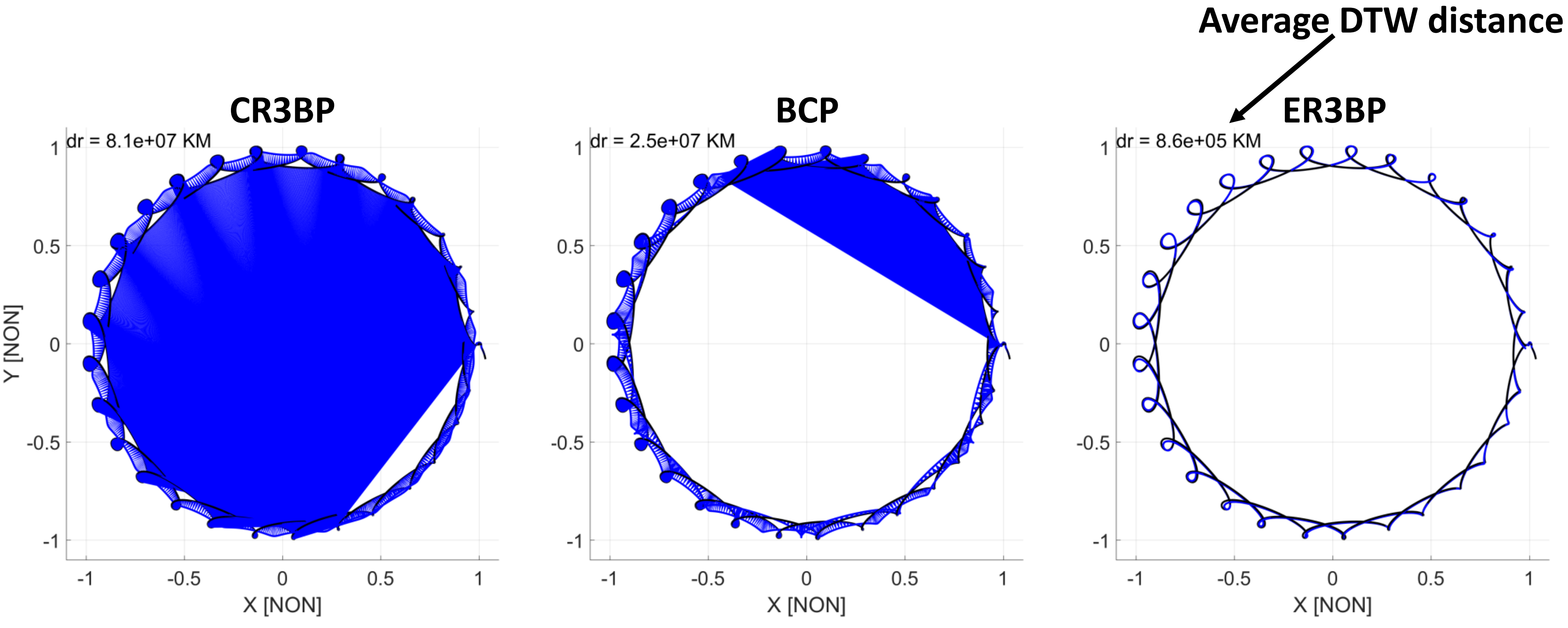
Results Geometry

- Repeat procedure for ER3BP model results
- **Asteroid trajectory**



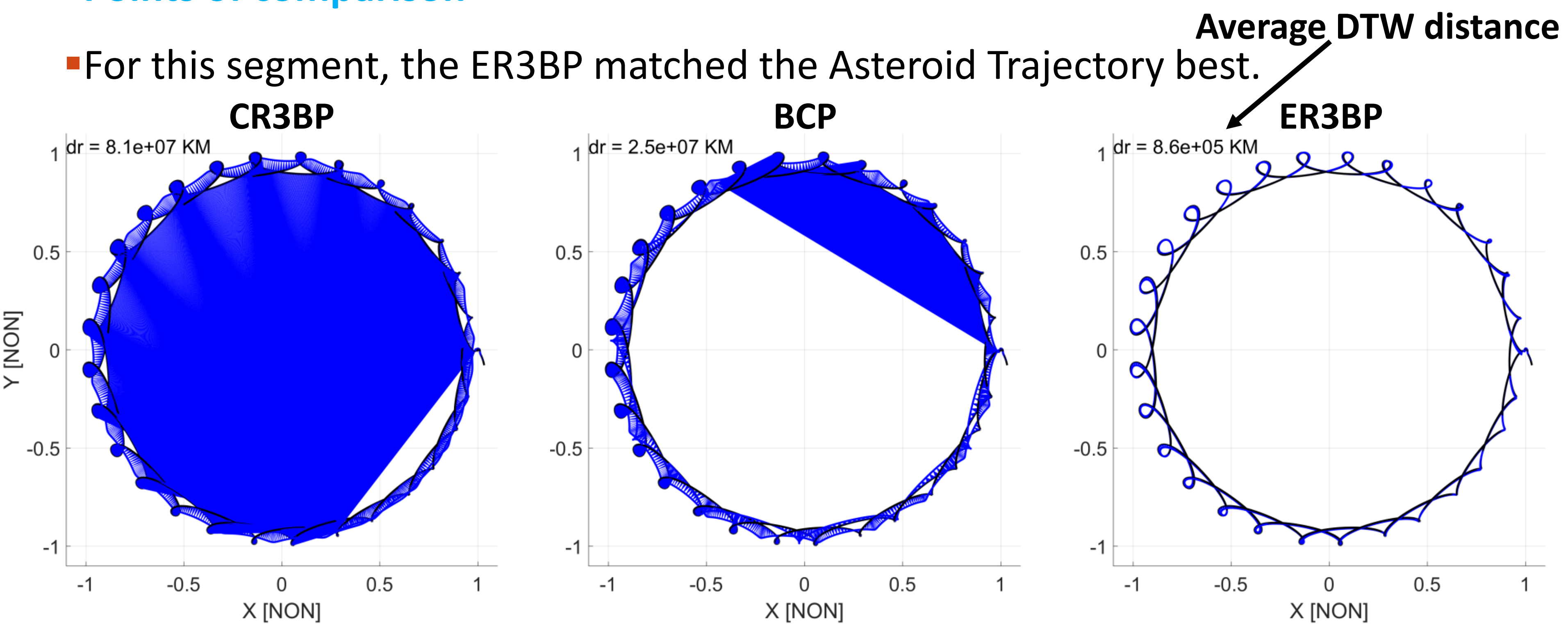
Results Geometry

- Repeat procedure for ER3BP model results
- **Asteroid trajectory**
- **Integrated trajectory**



Results Geometry

- Repeat procedure for ER3BP model results
- **Asteroid trajectory**
- **Integrated trajectory**
- **Points of comparison**
- For this segment, the ER3BP matched the Asteroid Trajectory best.



Result Summary

Computed parameters

- CR3BP is never the best model
- ER3BP best for segment 3
- BCP best for segment 1,2
- BCP models temp. capture dynamics well, even unadjusted

Tuned parameters

- CR3BP is never the best model
- ER3BP best for segment 1,3
  - Only requires small adjustment (~10 deg)
- BCP best for segment 2
- ER3BP nearly as good as BCP for segment 2
- Segment 2 required large adjustment for both BCP and ER3BP
  - BCP already good, small improvement
  - ER3BP large improvement

		Computed Parameters	Tuned Parameters	
Segment	Model	Mean DTW Distance [km]	Mean DTW Distance [km]	Angle Offset [deg]
1	CR3BP	8.1E+07		
	BCP	4.6E+07	2.5E+07	1.400
	ER3BP	1.2E+08	8.6E+05	-10.420
2	CR3BP	8.2E+05		
	BCP	7.4E+05	1.8E+05	89.388
	ER3BP	5.0E+06	2.2E+05	86.100
3	CR3BP	9.3E+07		
	BCP	9.3E+07	9.3E+07	
	ER3BP	6.4E+06	7.2E+05	-12.952

Conclusion

- BCP models temporary capture well using computed parameters
  - To be expected, higher fidelity of local perturbation
- ER3BP still models temporary capture phase well if slightly adjusted (see adjusted Seg. 1 & 3)
  - Indicates Lunar perturbation was not dominant
  - Temp. capture lasted 1 year, so effects of eccentricity non-negligible
- Eccentricity and Lunar perturbations approximately equivalent during temporary capture
  - Eccentricity effects increase due to length of temporary capture
  - Lunar effect decrease due to lack of close approaches
- Eccentricity dominates exterior and interior resonance phases (in Seg. 1 & 3)
  - To be expected, Earth-Moon can be approximated as combined point mass.
- Separations between models and reality are still large
  - Useful for global dynamical behavior analysis
  - Not useful for accurate determination of single particle

Future Work

- Compare ephemeris and simple model agreement for fictitious temporary capture asteroids (larger datasets)
  - Propagate in ephemeris model
  - Propagate in simple models
  - Compute similarity
- Produce combined BCP and ER3BP model that can model the majority of temporary capture objects well.

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California Institute of Technology

USC Viterbi

References

[1] Anderson, B. D., and Lo, M. W., “Dynamics of Asteroid 2006 RH120: Pre-Capture and Escape Phases,” 26th AAS/AIAA SpaceFlight Mechanics Meeting, Advances in the Astronautical Sciences, Vol. 158, Univelt, Inc., Napa, CA, 2016, pp. 777–795.

[2] Granvik, M., Vaubaillon, J., and Jedicke, R., “The population of natural Earth satellites,” Icarus, Vol. 218, No. 1, 2012, pp. 262–277. doi:10.1016/j.icarus.2011.12.003.

[3] Urrutxua, H., Scheeres, D., Bombardelli, C., Gonzalo, J. L., and Peláez, J., “What does it take to capture an Asteroid? A case study on capturing Asteroid 2006 RH120,” 24th AAS/AIAA Space Flight Mechanics Meeting, Advances in the Astronautical Sciences, Vol. 152, Santa Fe, NM, 2014, pp. 1117–1136.

[4] Sakoe, H., and Chiba, S., “Dynamic programming algorithm optimization for spoken word recognition,” IEEE transactions on acoustics, speech, and signal processing, Vol. 26, No. 1, 1978, pp. 43–49. doi:10.1109/tassp.1978.1163055.

[5] Eiter, T., and Mannila, H., “Computing discrete Fréchet distance,” Tech. rep., Tech. Report CD-TR 94/64, Information Systems Department, Technical University of Vienna, 1994.

[6] Alt, H., and Godau, M., “Computing the Fréchet distance between two polygonal curves,” International Journal of Computational Geometry & Applications, Vol. 5, No. 1–2, 1995, pp. 75–91. doi:10.1142/S0218195995000064.

[7] Koon, W. S., Lo, M. W., Marsden, J. E., and Ross, S. D., “Resonance and Capture of Jupiter Comets,” Celestial Mechanics and Dynamical Astronomy, Vol. 81, No. 1, 2001, pp. 27–38.

[8] Williams, G. V., “Distant Artificial Satellite Observation (DASO) Circular,” IAU Minor Planet Center, , No. 68, 2006.

[9] Szebehely, V., Theory of orbits: the restricted problem of three bodies, Academic Press, New York, 1967.

[10] Koon, W. S., Lo, M. W., Marsden, J. E., and Ross, S. D., “Heteroclinic connections between periodic orbits and resonance transitions in celestial mechanics,” Chaos: An Interdisciplinary Journal of Nonlinear Science, Vol. 10, No. 2, 2000, pp. 427–469. doi:10.1063/1.166509.

[11] Cronin, J., Richards, P. B., and Russell, L. H., “Some periodic solutions of a four-body problem,” Icarus, Vol. 3, No. 5-6, 1964, pp. 423–428. doi:10.1016/0019-1035(64)90003-X.

[12] Bate, R. R., Mueller, D. D., and White, J. E., Fundamentals of astrodynamics, Courier Corporation, 1971.

[13] Szebehely, V., and Giacaglia, G. E., “On the elliptic restricted problem of three bodies,” The Astronomical Journal, Vol. 69, No. 1, 1964, pp. 230–235.

[14] Folkner, W. M., Williams, J. G., Boggs, D. H., Park, R. S., and Kuchynka, P., “The planetary and lunar ephemerides DE430 and DE431,” Interplanet. Netw. Prog. Rep, Vol. 196, 2014, pp. 1–81.

[15] Anderson, B. D., and Lo, M. W., “NEO 2006 RH120 – Preliminary Resonance Analysis,” , 2014. JPL Interoffice Memorandum, NASA Center Innovation Fund Program at JPL.

[16] Mathworks, MATLAB and Signal Processing Toolbox Release 2017a, The Mathworks, Inc., Natick, MA, 2017.

[17] Ding, H., Trajcevski, G., Scheuermann, P., Wang, X., and Keogh, E., “Querying and mining of time series data: experimental comparison of representations and distance measures,” Proceedings of the VLDB Endowment, Vol. 1, No. 2, 2008, pp. 1542–1552. doi:10.14778/1454159.1454226.

Image Credit

- “Earth's "Other Moon“.” *Sky & Telescope*. 17 Apr. 2007. Web. 4 Aug. 2013. ‹ http://www.skyandtelescope.com/news/7067527.html ›.

Backup Slides

Complete Numerical Results

- Green boxes are the best match (smallest mean DTW distance)
- ER3BP orders of magnitude better than BCP for Seg. 1 & 3 when tuned
- BCP best in all cases for Seg. 2, but ER3BP very similar when tuned

		Computed Parameters			Tuned Parameters			
Segment	**Model**	**Forw. [km]**	**Back. [km]**	**Overall [km]**	**Forw. [km]**	**Back. [km]**	**Overall [km]**	**Angle Offset [deg]**
1	**CR3BP**		8.1E+07	8.1E+07				
	BCP		4.6E+07	4.6E+07		2.5E+07	2.5E+07	1.400
	ER3BP		1.2E+08	1.2E+08		8.6E+05	8.6E+05	-10.420
2	**CR3BP**	2.3E+05	1.4E+06	8.2E+05				
	BCP	1.5E+05	1.3E+06	7.4E+05	1.0E+05	2.6E+05	1.8E+05	89.388
	ER3BP	7.7E+05	9.3E+06	5.0E+06	1.5E+05	2.9E+05	2.2E+05	86.100
3	**CR3BP**	9.3E+07		9.3E+07				
	BCP	9.3E+07		9.3E+07	9.3E+07		9.3E+07	
	ER3BP	6.4E+06		6.4E+06	7.2E+05		7.2E+05	-12.952

Complete Numerical Results

- Green boxes are the best match (smallest mean DTW distance)
- ER3BP orders of magnitude better than BCP for Seg. 1 & 3 when tuned
- BCP best in all cases for Seg. 2, but ER3BP very similar when tuned

		Computed Parameters	Tuned Parameters	
Segment	**Model**	**Mean DTW Distance [km]**	**Mean DTW Distance [km]**	**Angle Offset [deg]**
1	**CR3BP**	8.1E+07		
	BCP	4.6E+07	2.5E+07	1.400
	ER3BP	1.2E+08	8.6E+05	-10.420
2	**CR3BP**	8.2E+05		
	BCP	7.4E+05	1.8E+05	89.388
	ER3BP	5.0E+06	2.2E+05	86.100
3	**CR3BP**	9.3E+07		
	BCP	9.3E+07	9.3E+07	
	ER3BP	6.4E+06	7.2E+05	-12.952

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Result Summary

Computed parameters

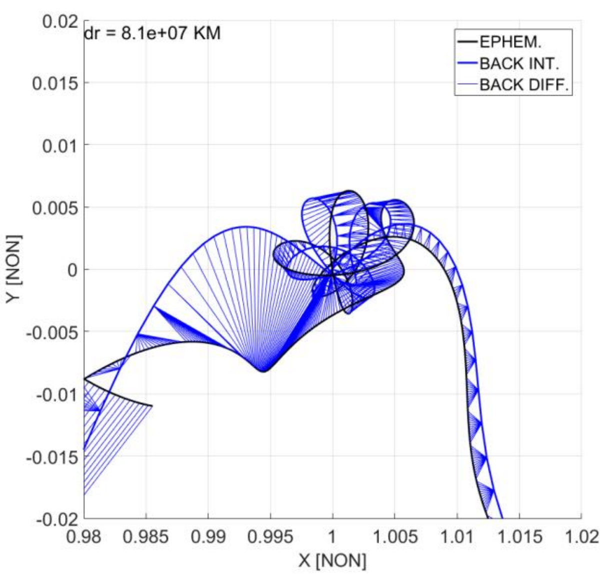
- CR3BP is never the best model
- ER3BP best for segment 3
- BCP best for segment 1,2
- BCP models temp. capture dynamics well, even unadjusted

Tuned parameters

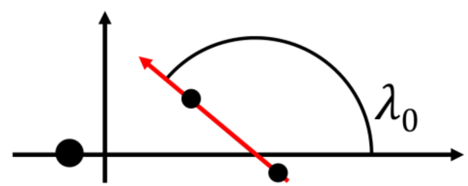
- CR3BP is never the best model
- ER3BP best for segment 1,3
  - Only requires small adjustment (~10 deg)
- BCP best for segment 2
- ER3BP nearly as good as BCP for segment 2
- Segment 2 required large adjustment for both BCP and ER3BP
  - BCP already good, small improvement
  - ER3BP large improvement

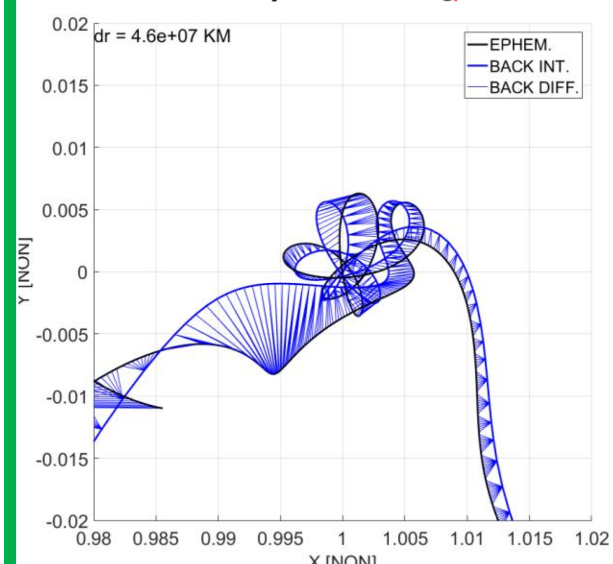
		Computed Parameters			Tuned Parameters			
Segment	Model	Forw. [km]	Back. [km]	Overall [km]	Forw. [km]	Back. [km]	Overall [km]	Angle Offset [deg]
1	CR3BP		8.1E+07	8.1E+07				
	BCP		4.6E+07	4.6E+07		2.5E+07	2.5E+07	1.400
	ER3BP		1.2E+08	1.2E+08		8.6E+05	8.6E+05	-10.420
2	CR3BP	2.3E+05	1.4E+06	8.2E+05				
	BCP	1.5E+05	1.3E+06	7.4E+05	1.0E+05	2.6E+05	1.8E+05	89.388
	ER3BP	7.7E+05	9.3E+06	5.0E+06	1.5E+05	2.9E+05	2.2E+05	86.100
3	CR3BP	9.3E+07		9.3E+07				
	BCP	9.3E+07		9.3E+07	9.3E+07		9.3E+07	
	ER3BP	6.4E+06		6.4E+06	7.2E+05		7.2E+05	-12.952

CR3BP

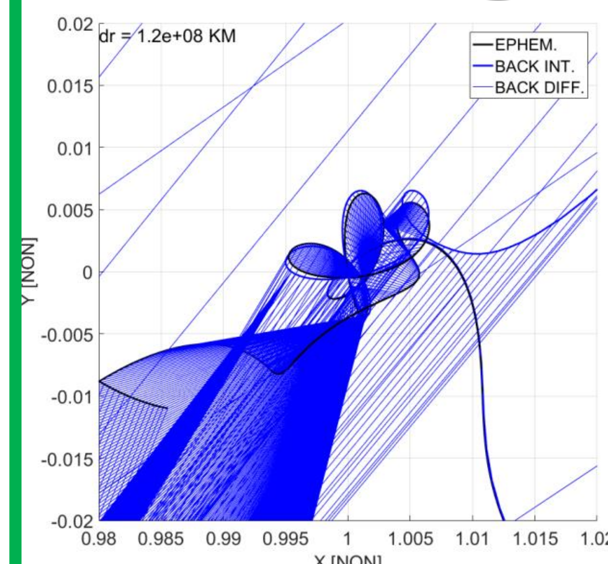


BCP





ER3BP



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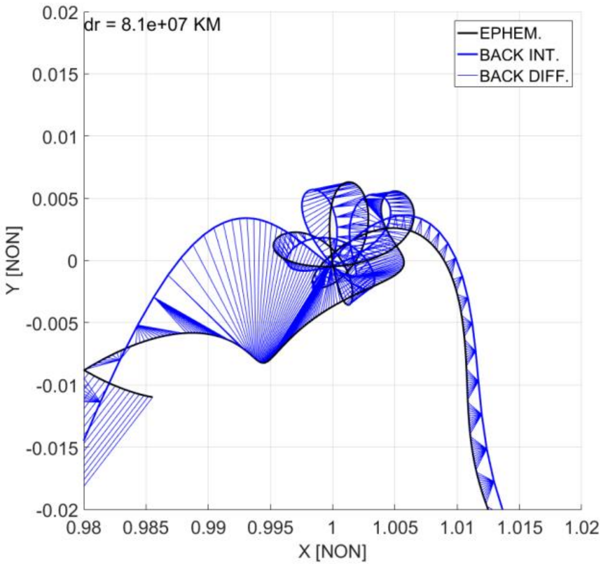
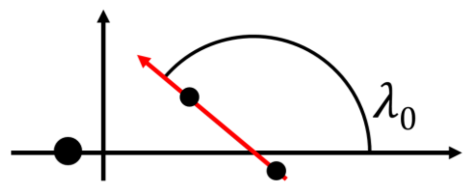
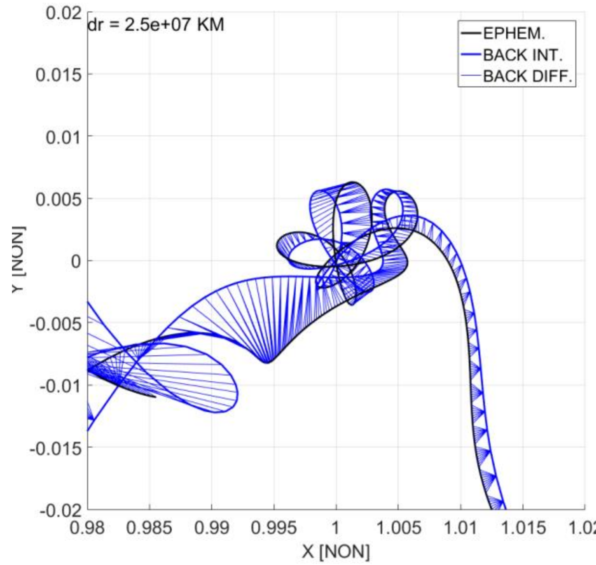
		Computed Parameters			Tuned Parameters			
Segment	**Model**	**Forw. [km]**	**Back. [km]**	**Overall [km]**	**Forw. [km]**	**Back. [km]**	**Overall [km]**	**Angle Offset [deg]**
1	**CR3BP**		8.1E+07	8.1E+07				
	BCP		4.6E+07	4.6E+07		2.5E+07	2.5E+07	1.400
	ER3BP		1.2E+08	1.2E+08		8.6E+05	8.6E+05	-10.420
2	**CR3BP**	2.3E+05	1.4E+06	8.2E+05				
	BCP	1.5E+05	1.3E+06	7.4E+05	1.0E+05	2.6E+05	1.8E+05	89.388
	ER3BP	7.7E+05	9.3E+06	5.0E+06	1.5E+05	2.9E+05	2.2E+05	86.100
3	**CR3BP**	9.3E+07		9.3E+07				
	BCP	9.3E+07		9.3E+07	9.3E+07		9.3E+07	
	ER3BP	6.4E+06		6.4E+06	7.2E+05		7.2E+05	-12.952

CR3BP

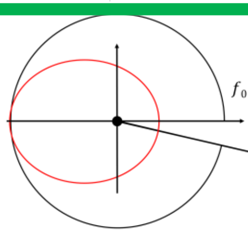


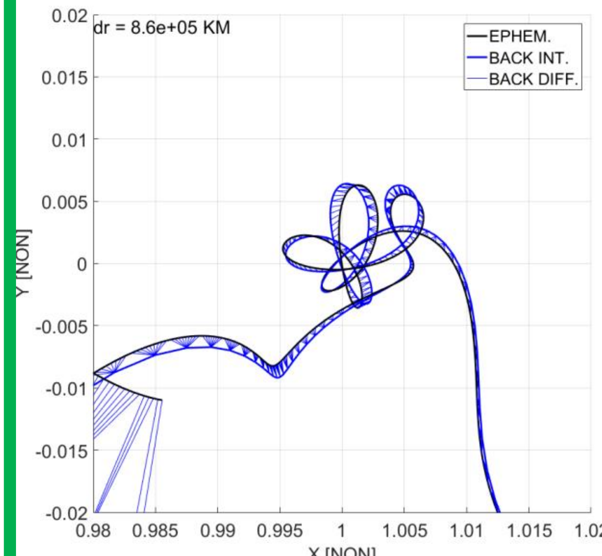
BCP



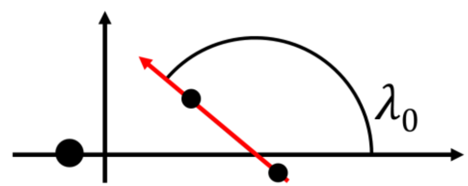


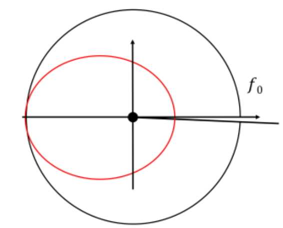
ER3BP



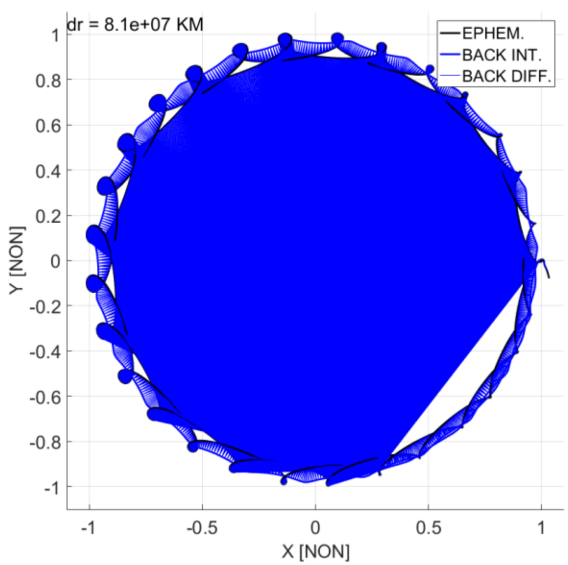


		Computed Parameters			Tuned Parameters			
Segment	Model	Forw. [km]	Back. [km]	Overall [km]	Forw. [km]	Back. [km]	Overall [km]	Angle Offset [deg]
1	CR3BP		8.1E+07	8.1E+07				
	BCP		4.6E+07	4.6E+07		2.5E+07	2.5E+07	1.400
	ER3BP		1.2E+08	1.2E+08		8.6E+05	8.6E+05	-10.420
2	CR3BP	2.3E+05	1.4E+06	8.2E+05				
	BCP	1.5E+05	1.3E+06	7.4E+05	1.0E+05	2.6E+05	1.8E+05	89.388
	ER3BP	7.7E+05	9.3E+06	5.0E+06	1.5E+05	2.9E+05	2.2E+05	86.100
3	CR3BP	9.3E+07		9.3E+07				
	BCP	9.3E+07		9.3E+07	9.3E+07		9.3E+07	
	ER3BP	6.4E+06		6.4E+06	7.2E+05		7.2E+05	-12.952

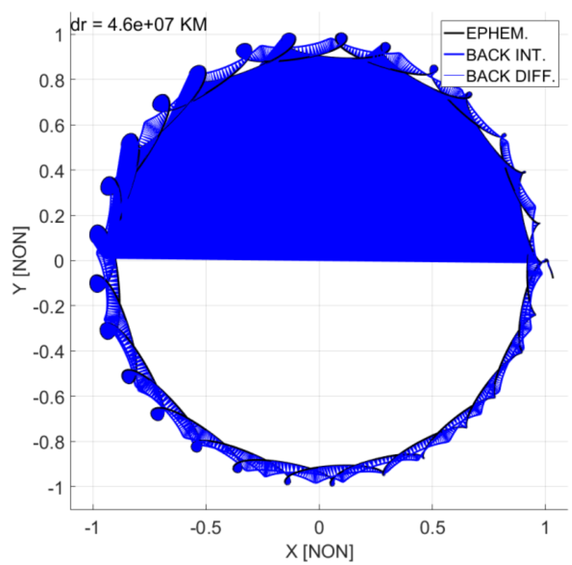




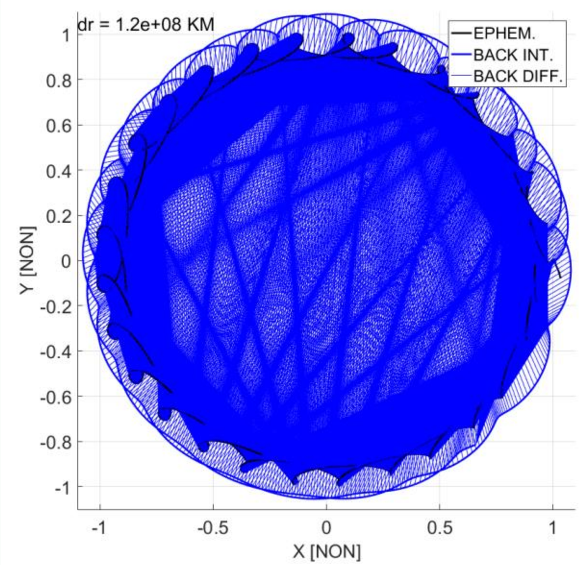
CR3BP



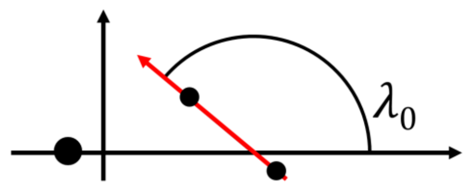
BCP

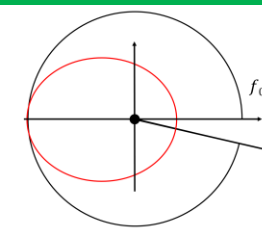


ER3BP

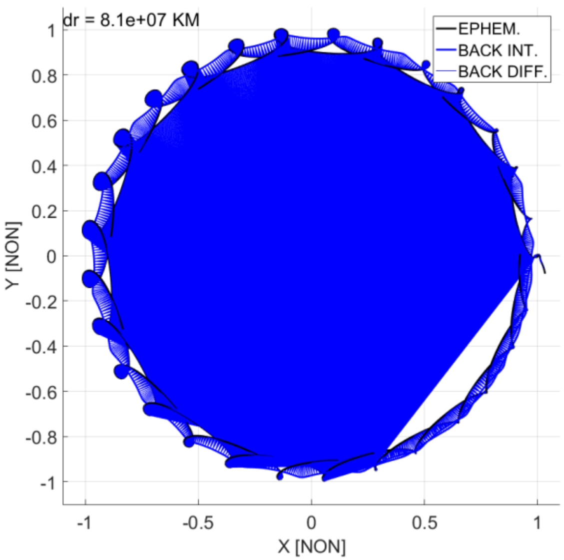


		Computed Parameters			Tuned Parameters			
Segment	Model	Forw. [km]	Back. [km]	Overall [km]	Forw. [km]	Back. [km]	Overall [km]	Angle Offset [deg]
1	CR3BP		8.1E+07	8.1E+07				
	BCP		4.6E+07	4.6E+07		2.5E+07	2.5E+07	1.400
	ER3BP		1.2E+08	1.2E+08		8.6E+05	8.6E+05	-10.420
2	CR3BP	2.3E+05	1.4E+06	8.2E+05				
	BCP	1.5E+05	1.3E+06	7.4E+05	1.0E+05	2.6E+05	1.8E+05	89.388
	ER3BP	7.7E+05	9.3E+06	5.0E+06	1.5E+05	2.9E+05	2.2E+05	86.100
3	CR3BP	9.3E+07		9.3E+07				
	BCP	9.3E+07		9.3E+07	9.3E+07		9.3E+07	
	ER3BP	6.4E+06		6.4E+06	7.2E+05		7.2E+05	-12.952

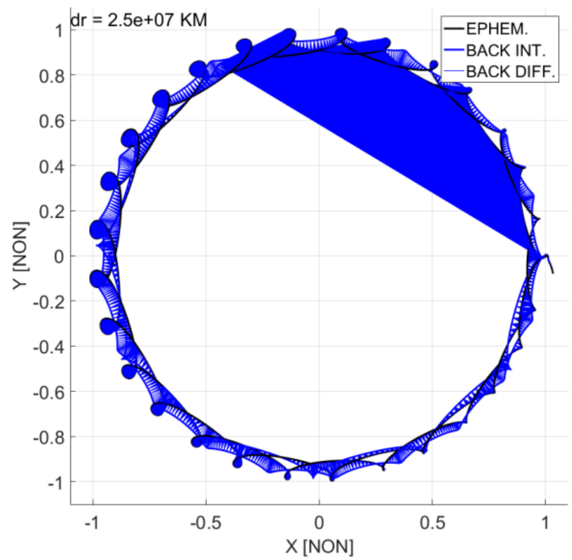




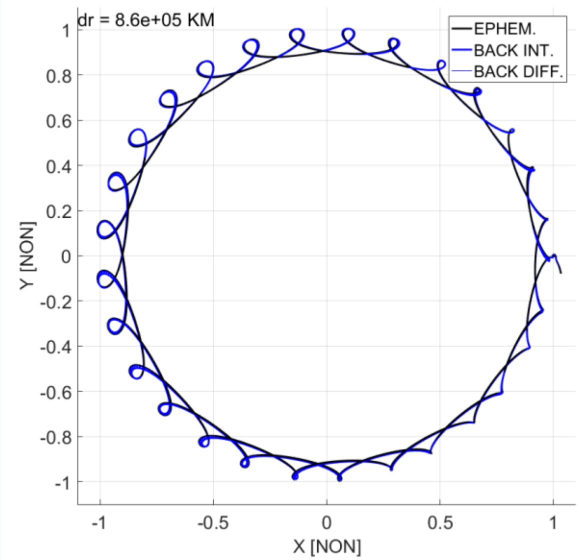
CR3BP



BCP

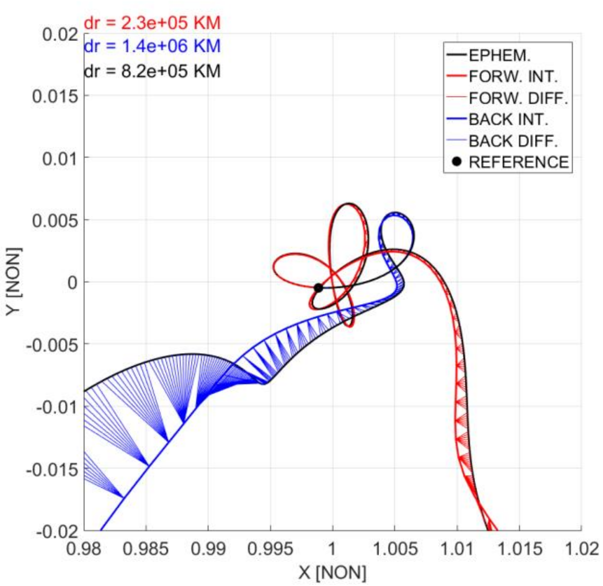


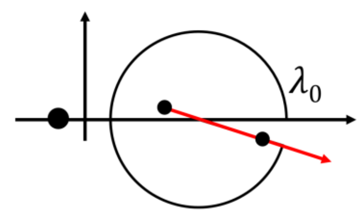
ER3BP



Segment	Model	Computed Parameters: Forw. [km]	Computed Parameters: Back. [km]	Computed Parameters: Overall [km]	Tuned Parameters: Forw. [km]	Tuned Parameters: Back. [km]	Tuned Parameters: Overall [km]	Angle Offset [deg]
1	CR3BP		8.1E+07	8.1E+07				
	BCP		4.6E+07	4.6E+07		2.5E+07	2.5E+07	1.400
	ER3BP		1.2E+08	1.2E+08		8.6E+05	8.6E+05	-10.420
2	CR3BP	2.3E+05	1.4E+06	8.2E+05				
	BCP	1.5E+05	1.3E+06	7.4E+05	1.0E+05	2.6E+05	1.8E+05	89.388
	ER3BP	7.7E+05	9.3E+06	5.0E+06	1.5E+05	2.9E+05	2.2E+05	86.100
3	CR3BP	9.3E+07		9.3E+07				
	BCP	9.3E+07		9.3E+07	9.3E+07		9.3E+07	
	ER3BP	6.4E+06		6.4E+06	7.2E+05		7.2E+05	-12.952

CR3BP





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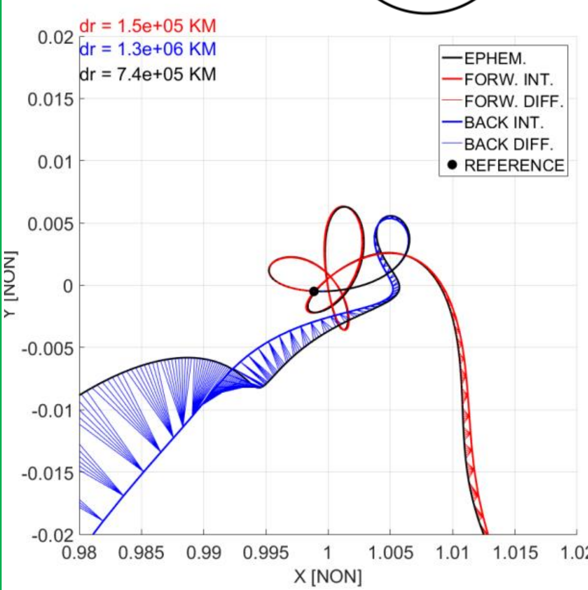
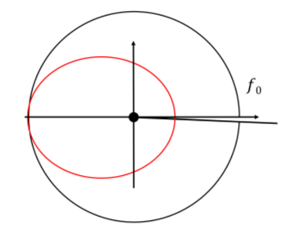
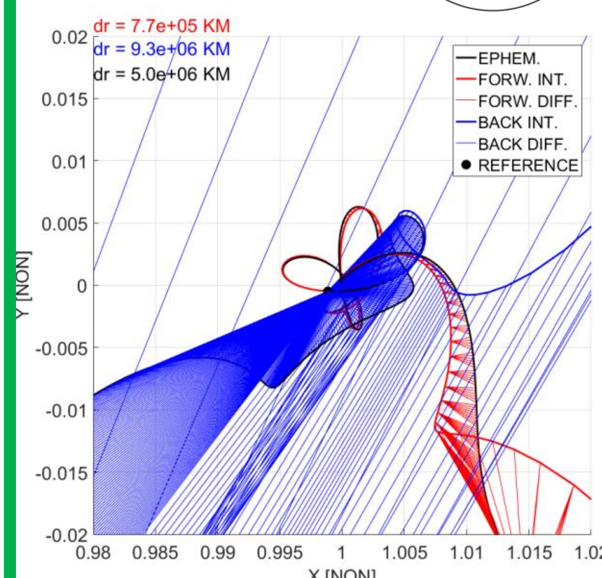


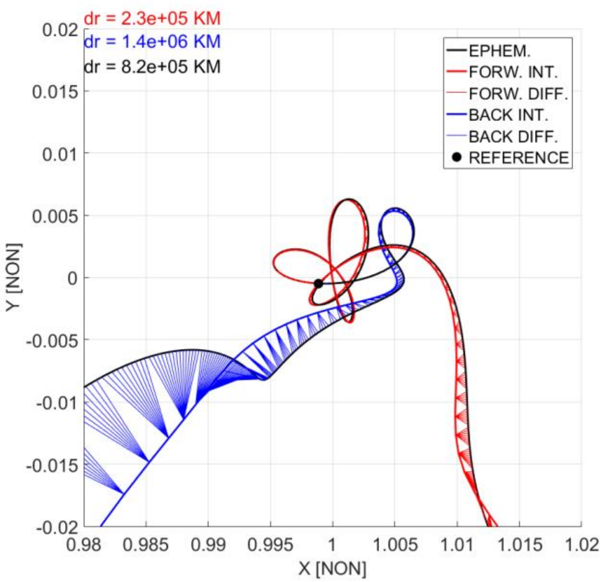


ER3BP

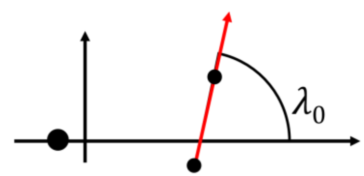


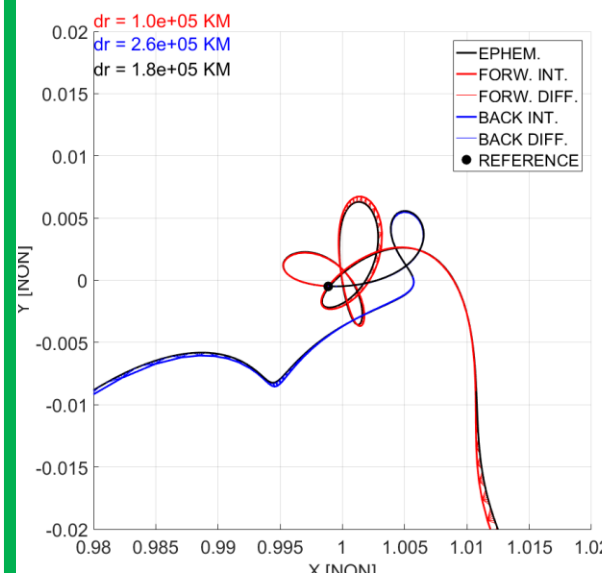
Segment	Model	Computed Parameters			Tuned Parameters			
		Forw. [km]	Back. [km]	Overall [km]	Forw. [km]	Back. [km]	Overall [km]	Angle Offset [deg]
1	CR3BP		8.1E+07	8.1E+07				
	BCP		4.6E+07	4.6E+07		2.5E+07	2.5E+07	1.400
	ER3BP		1.2E+08	1.2E+08		8.6E+05	8.6E+05	-10.420
2	CR3BP	2.3E+05	1.4E+06	8.2E+05				
	BCP	1.5E+05	1.3E+06	7.4E+05	1.0E+05	2.6E+05	1.8E+05	89.388
	ER3BP	7.7E+05	9.3E+06	5.0E+06	1.5E+05	2.9E+05	2.2E+05	86.100
3	CR3BP	9.3E+07		9.3E+07				
	BCP	9.3E+07		9.3E+07	9.3E+07		9.3E+07	
	ER3BP	6.4E+06		6.4E+06	7.2E+05		7.2E+05	-12.952

CR3BP



BCP





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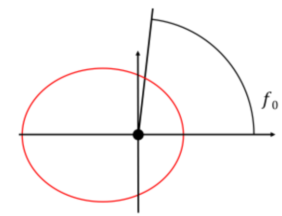
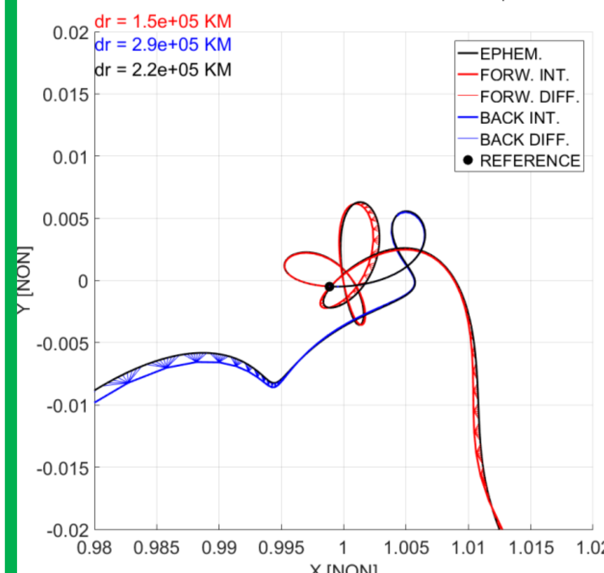
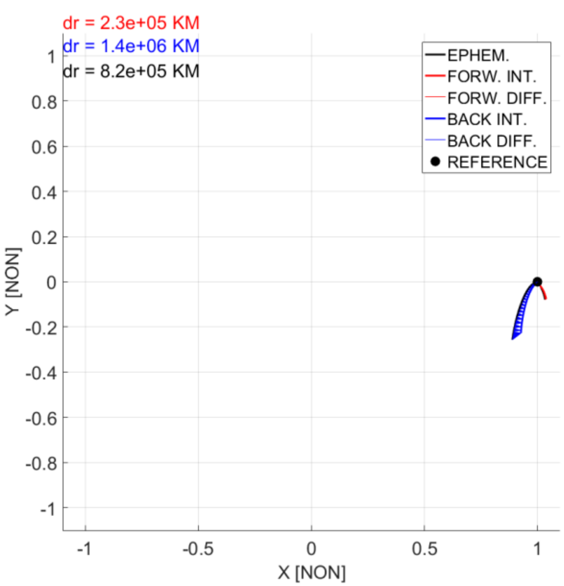


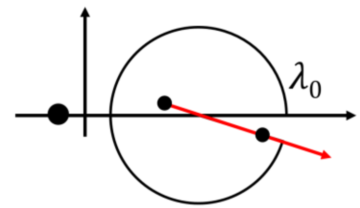


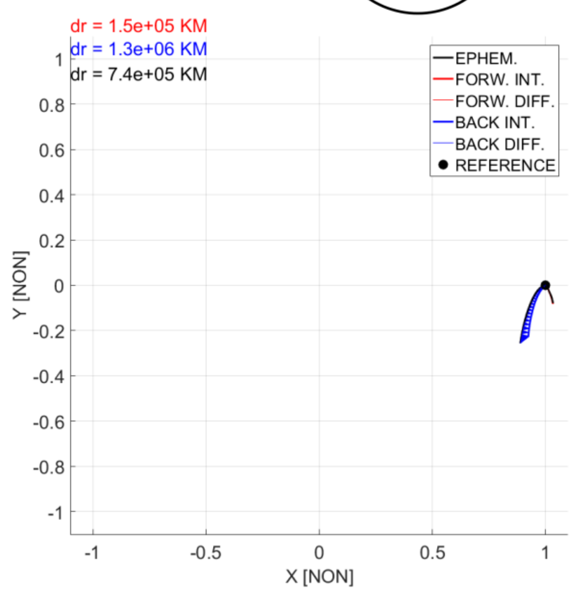
		Computed Parameters			Tuned Parameters			
Segment	**Model**	**Forw. [km]**	**Back. [km]**	**Overall [km]**	**Forw. [km]**	**Back. [km]**	**Overall [km]**	**Angle Offset [deg]**
1	**CR3BP**		8.1E+07	8.1E+07				
	BCP		4.6E+07	4.6E+07		2.5E+07	2.5E+07	1.400
	ER3BP		1.2E+08	1.2E+08		8.6E+05	8.6E+05	-10.420
2	**CR3BP**	2.3E+05	1.4E+06	8.2E+05				
	BCP	1.5E+05	1.3E+06	7.4E+05	1.0E+05	2.6E+05	1.8E+05	89.388
	ER3BP	7.7E+05	9.3E+06	5.0E+06	1.5E+05	2.9E+05	2.2E+05	86.100
3	**CR3BP**	9.3E+07		9.3E+07				
	BCP	9.3E+07		9.3E+07	9.3E+07		9.3E+07	
	ER3BP	6.4E+06		6.4E+06	7.2E+05		7.2E+05	-12.952

CR3BP

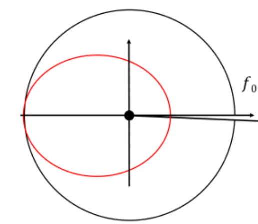


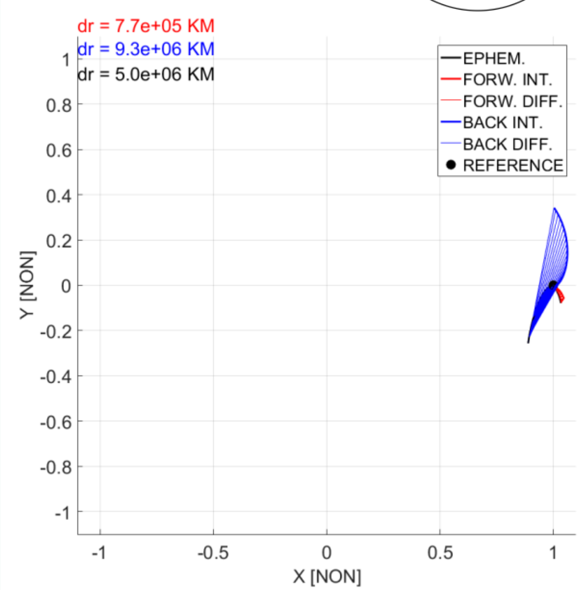
BCP





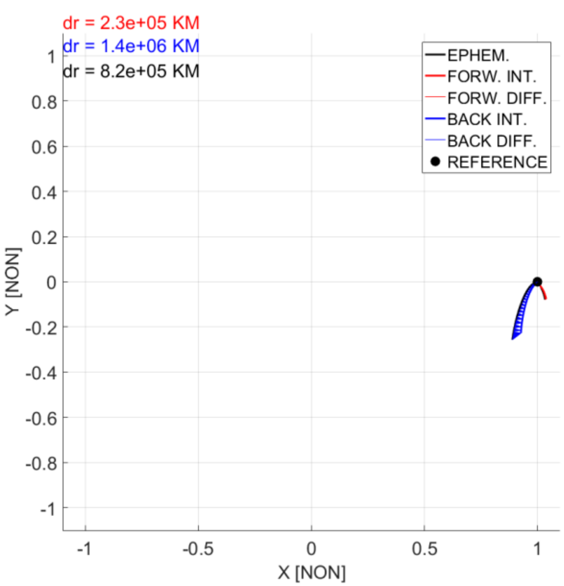
ER3BP



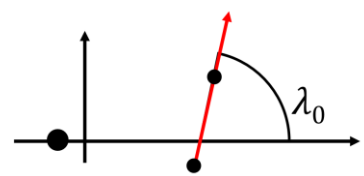


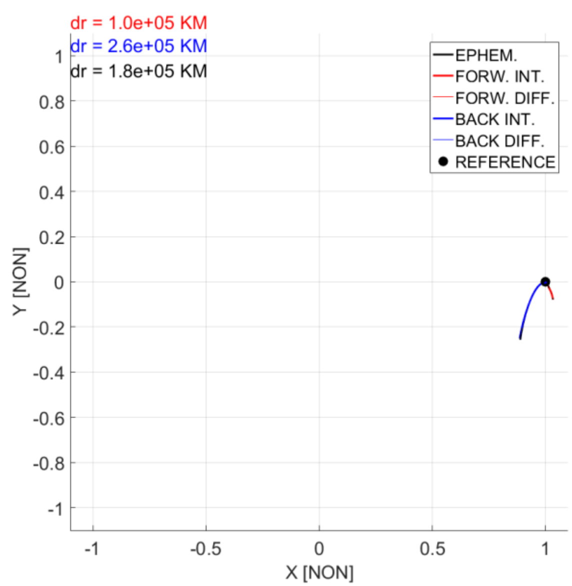
		Computed Parameters			Tuned Parameters			
Segment	**Model**	**Forw. [km]**	**Back. [km]**	**Overall [km]**	**Forw. [km]**	**Back. [km]**	**Overall [km]**	**Angle Offset [deg]**
1	**CR3BP**		8.1E+07	8.1E+07				
	BCP		4.6E+07	4.6E+07		2.5E+07	2.5E+07	1.400
	ER3BP		1.2E+08	1.2E+08		8.6E+05	8.6E+05	-10.420
2	**CR3BP**	2.3E+05	1.4E+06	8.2E+05				
	BCP	1.5E+05	1.3E+06	7.4E+05	1.0E+05	2.6E+05	1.8E+05	89.388
	ER3BP	7.7E+05	9.3E+06	5.0E+06	1.5E+05	2.9E+05	2.2E+05	86.100
3	**CR3BP**	9.3E+07		9.3E+07				
	BCP	9.3E+07		9.3E+07	9.3E+07		9.3E+07	
	ER3BP	6.4E+06		6.4E+06	7.2E+05		7.2E+05	-12.952

CR3BP

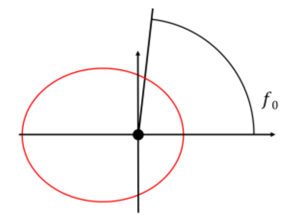


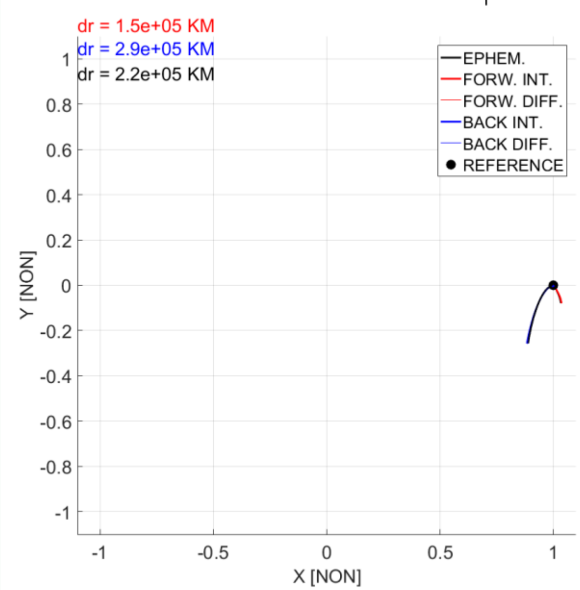
BCP



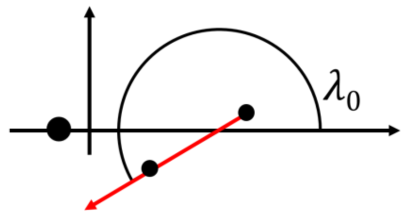


ER3BP

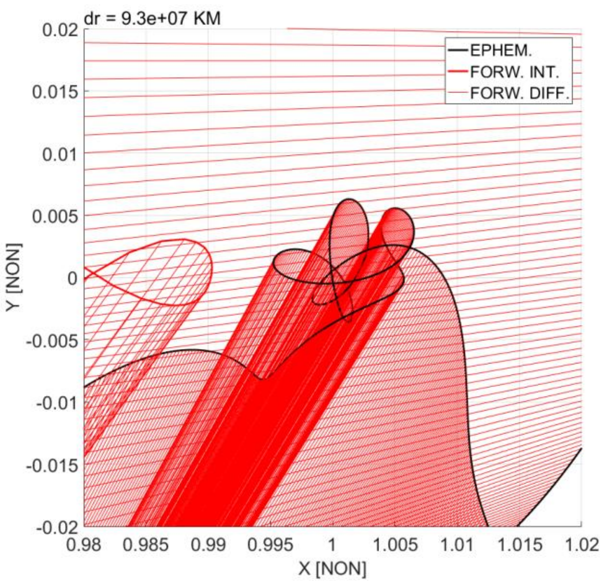




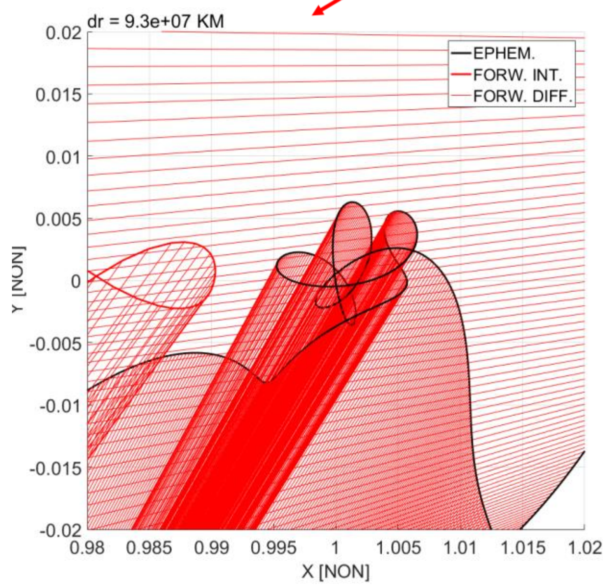
		Computed Parameters			Tuned Parameters			
Segment	**Model**	**Forw. [km]**	**Back. [km]**	**Overall [km]**	**Forw. [km]**	**Back. [km]**	**Overall [km]**	**Angle Offset [deg]**
1	**CR3BP**		8.1E+07	8.1E+07				
	BCP		4.6E+07	4.6E+07		2.5E+07	2.5E+07	1.400
	ER3BP		1.2E+08	1.2E+08		8.6E+05	8.6E+05	-10.420
2	**CR3BP**	2.3E+05	1.4E+06	8.2E+05				
	BCP	1.5E+05	1.3E+06	7.4E+05	1.0E+05	2.6E+05	1.8E+05	89.388
	ER3BP	7.7E+05	9.3E+06	5.0E+06	1.5E+05	2.9E+05	2.2E+05	86.100
3	**CR3BP**	9.3E+07		9.3E+07				
	BCP	9.3E+07		9.3E+07	9.3E+07		9.3E+07	
	ER3BP	6.4E+06		6.4E+06	7.2E+05		7.2E+05	-12.952



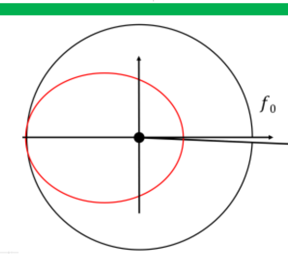
CR3BP

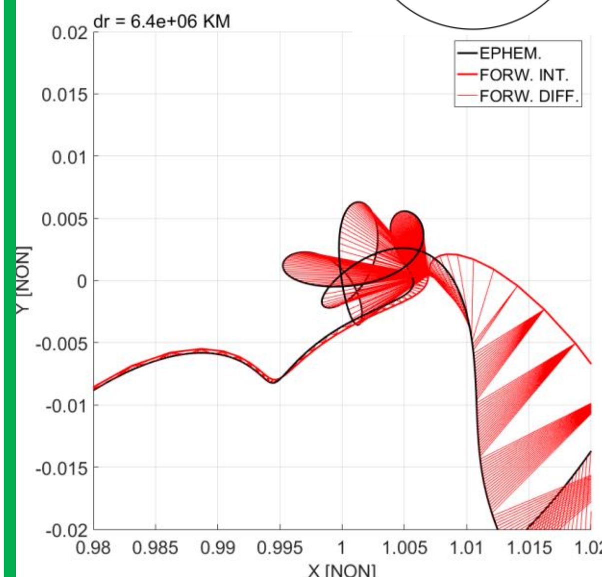


BCP

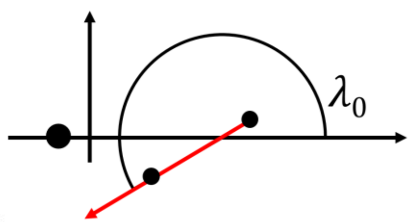


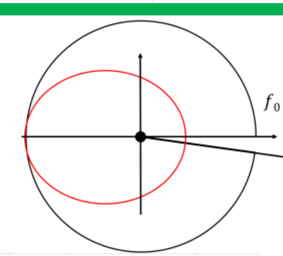
ER3BP



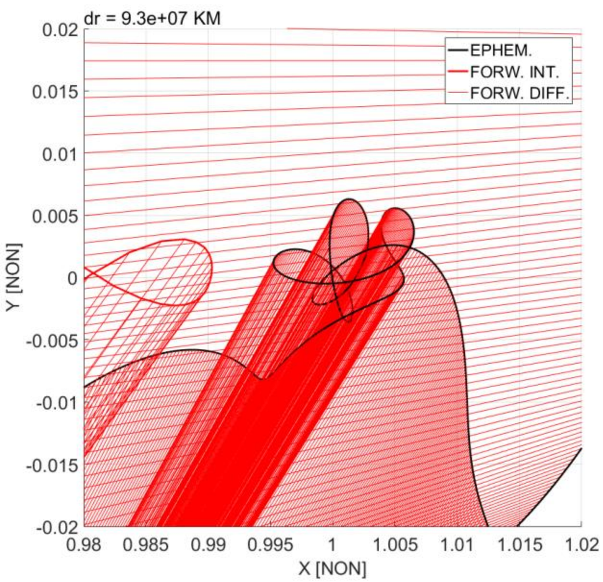


Segment	Model	Computed Parameters: Forw. [km]	Computed Parameters: Back. [km]	Computed Parameters: Overall [km]	Tuned Parameters: Forw. [km]	Tuned Parameters: Back. [km]	Tuned Parameters: Overall [km]	Tuned Parameters: Angle Offset [deg]
1	CR3BP		8.1E+07	8.1E+07				
	BCP		4.6E+07	4.6E+07		2.5E+07	2.5E+07	1.400
	ER3BP		1.2E+08	1.2E+08		8.6E+05	8.6E+05	-10.420
2	CR3BP	2.3E+05	1.4E+06	8.2E+05				
	BCP	1.5E+05	1.3E+06	7.4E+05	1.0E+05	2.6E+05	1.8E+05	89.388
	ER3BP	7.7E+05	9.3E+06	5.0E+06	1.5E+05	2.9E+05	2.2E+05	86.100
3	CR3BP	9.3E+07		9.3E+07				
	BCP	9.3E+07		9.3E+07	9.3E+07		9.3E+07	
	ER3BP	6.4E+06		6.4E+06	7.2E+05		7.2E+05	-12.952

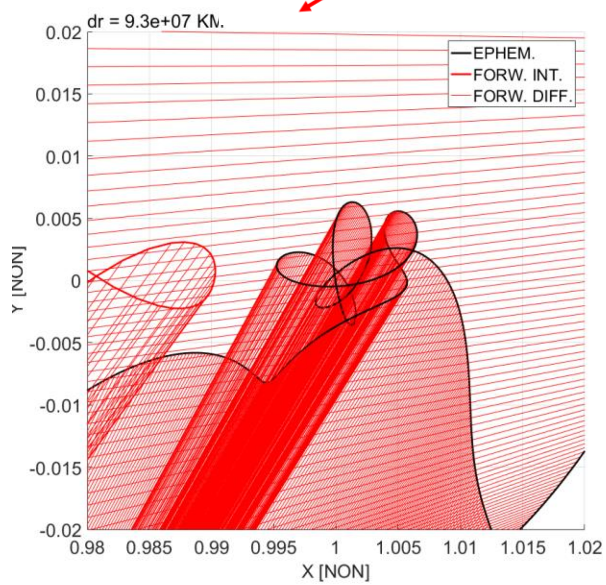




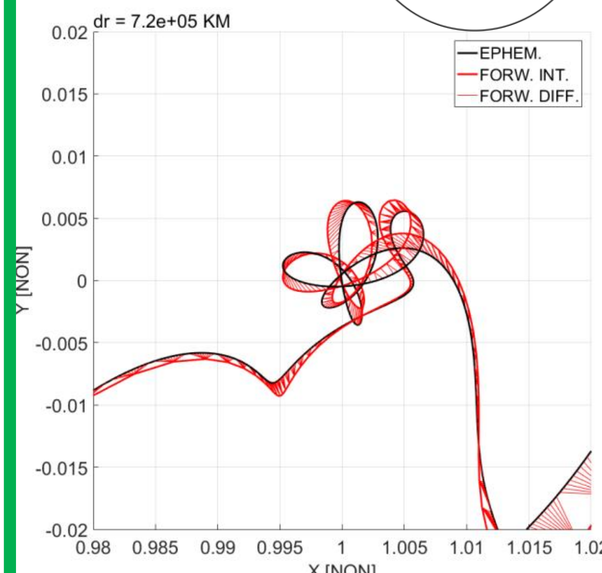
CR3BP



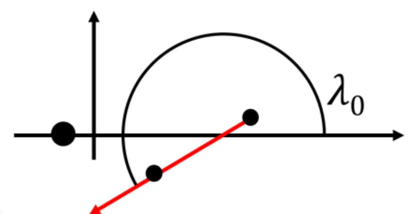
BCP

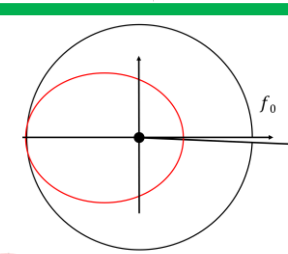


ER3BP

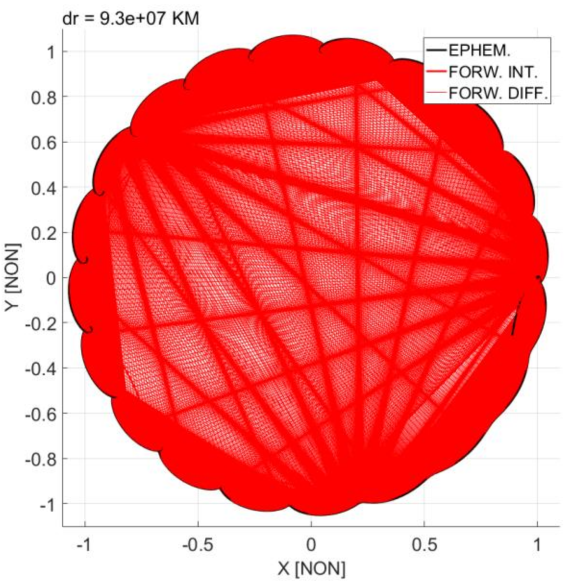


		Computed Parameters			Tuned Parameters			
Segment	**Model**	**Forw. [km]**	**Back. [km]**	**Overall [km]**	**Forw. [km]**	**Back. [km]**	**Overall [km]**	**Angle Offset [deg]**
1	**CR3BP**		8.1E+07	8.1E+07				
	BCP		4.6E+07	4.6E+07		2.5E+07	2.5E+07	1.400
	ER3BP		1.2E+08	1.2E+08		8.6E+05	8.6E+05	-10.420
2	**CR3BP**	2.3E+05	1.4E+06	8.2E+05				
	BCP	1.5E+05	1.3E+06	7.4E+05	1.0E+05	2.6E+05	1.8E+05	89.388
	ER3BP	7.7E+05	9.3E+06	5.0E+06	1.5E+05	2.9E+05	2.2E+05	86.100
3	**CR3BP**	9.3E+07		9.3E+07				
	BCP	9.3E+07		9.3E+07	9.3E+07		9.3E+07	
	ER3BP	6.4E+06		6.4E+06	7.2E+05		7.2E+05	-12.952

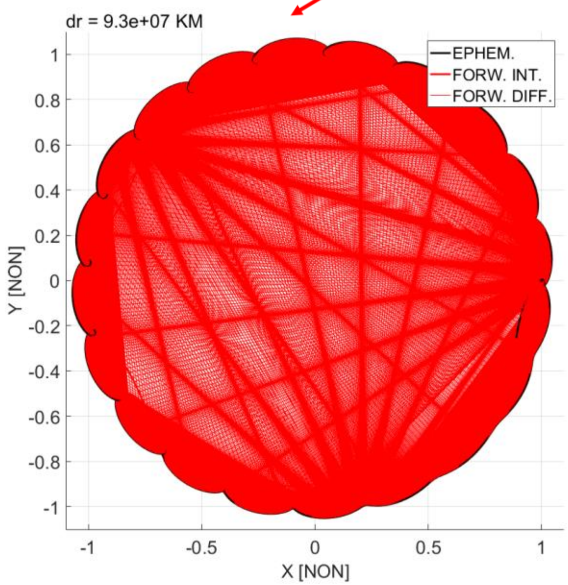




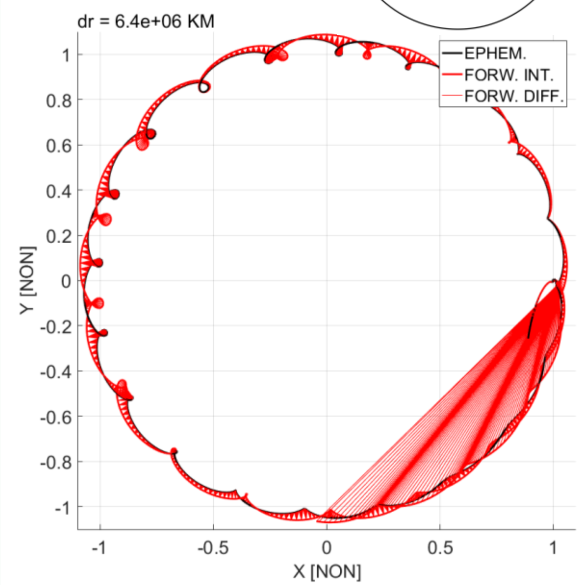
CR3BP



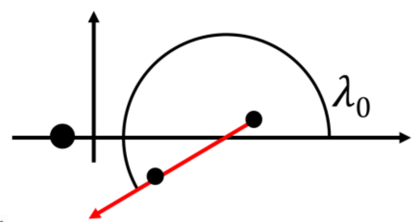
BCP

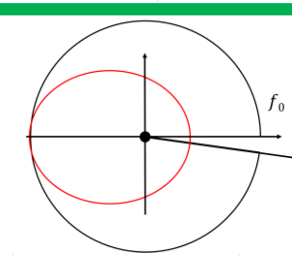


ER3BP

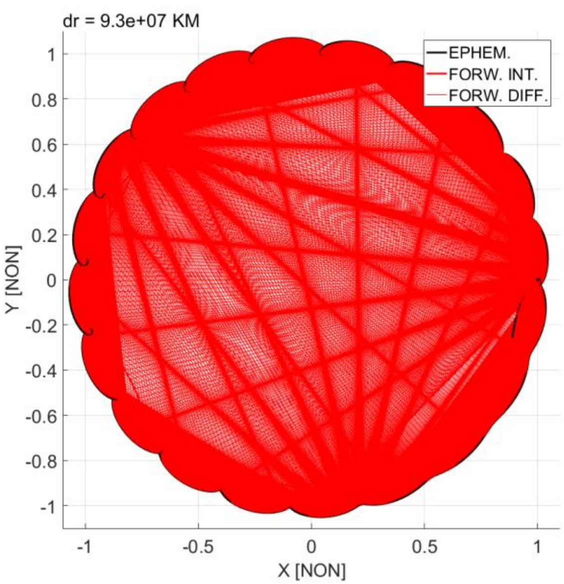


Segment	Model	Computed Parameters: Forw. [km]	Computed Parameters: Back. [km]	Computed Parameters: Overall [km]	Tuned Parameters: Forw. [km]	Tuned Parameters: Back. [km]	Tuned Parameters: Overall [km]	Angle Offset [deg]
1	CR3BP		8.1E+07	8.1E+07				
	BCP		4.6E+07	4.6E+07		2.5E+07	2.5E+07	1.400
	ER3BP		1.2E+08	1.2E+08		8.6E+05	8.6E+05	-10.420
2	CR3BP	2.3E+05	1.4E+06	8.2E+05				
	BCP	1.5E+05	1.3E+06	7.4E+05	1.0E+05	2.6E+05	1.8E+05	89.388
	ER3BP	7.7E+05	9.3E+06	5.0E+06	1.5E+05	2.9E+05	2.2E+05	86.100
3	CR3BP	9.3E+07		9.3E+07				
	BCP	9.3E+07		9.3E+07	9.3E+07		9.3E+07	
	ER3BP	6.4E+06		6.4E+06	7.2E+05		7.2E+05	-12.952

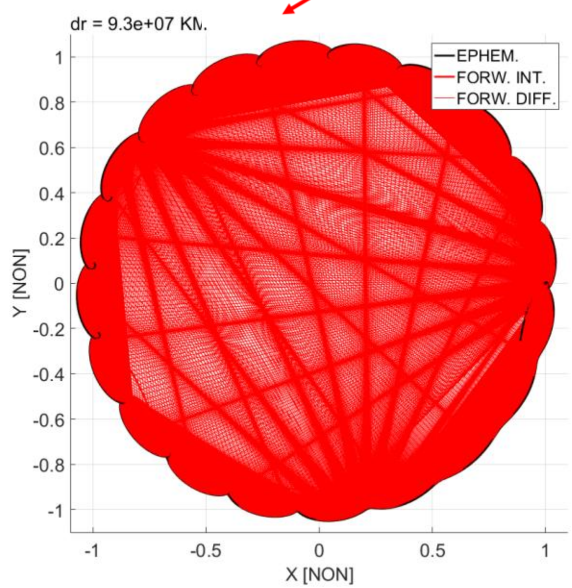




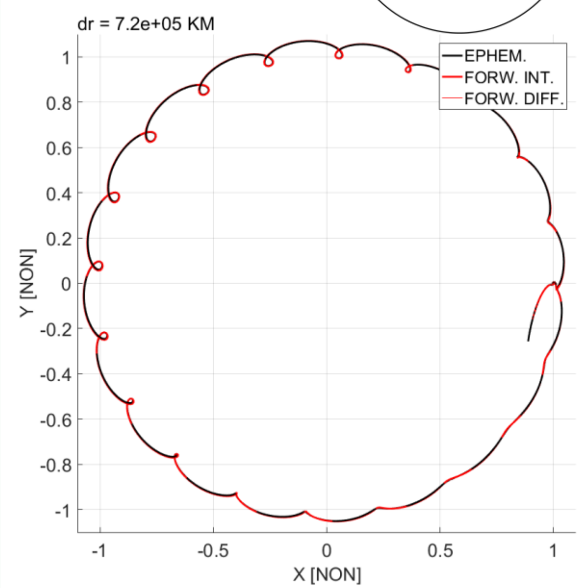
CR3BP



BCP

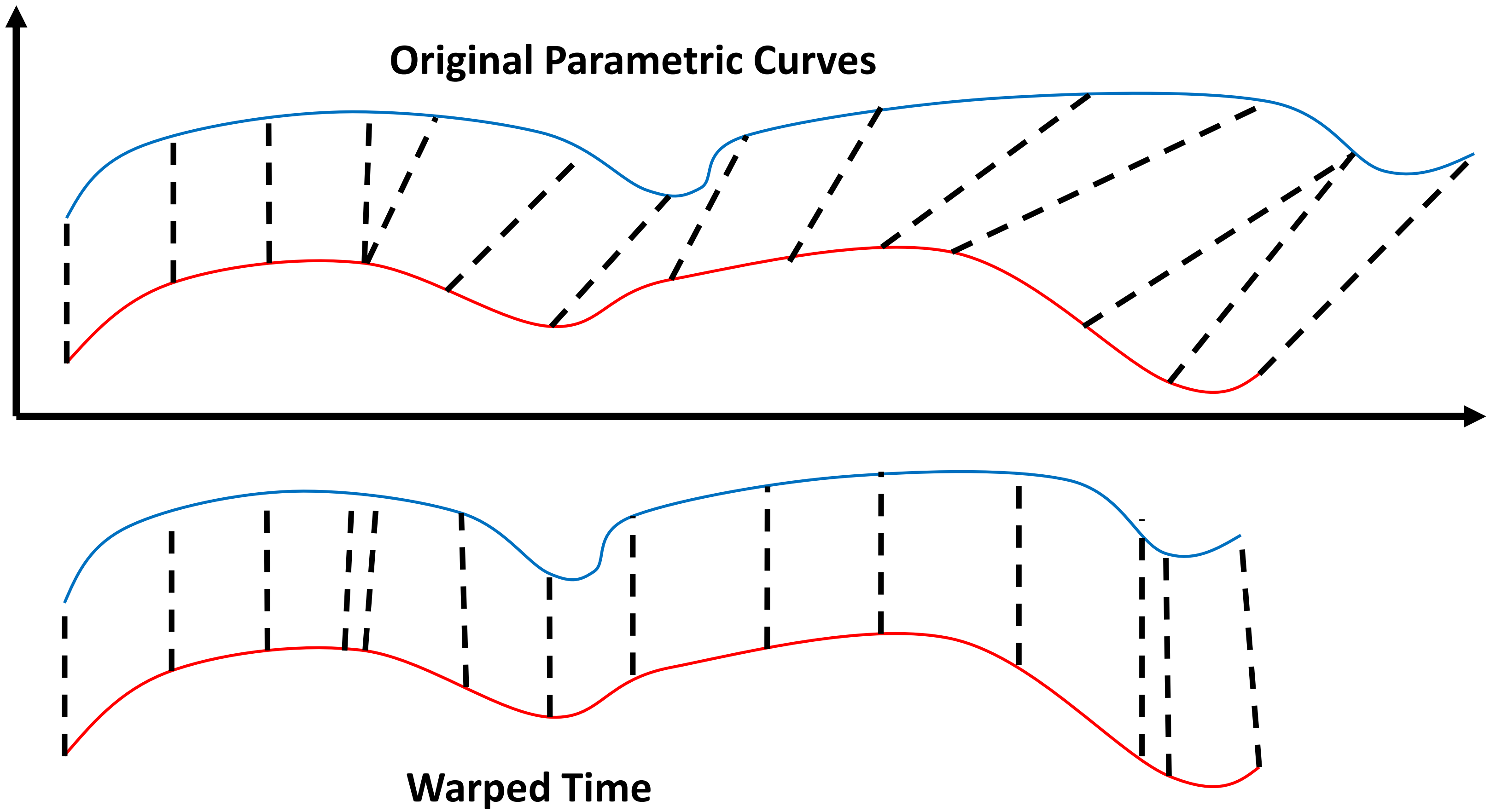


ER3BP



Dynamic Time Warping

- How similar is curve a to curve b?
- Shape only, ignore time stretching



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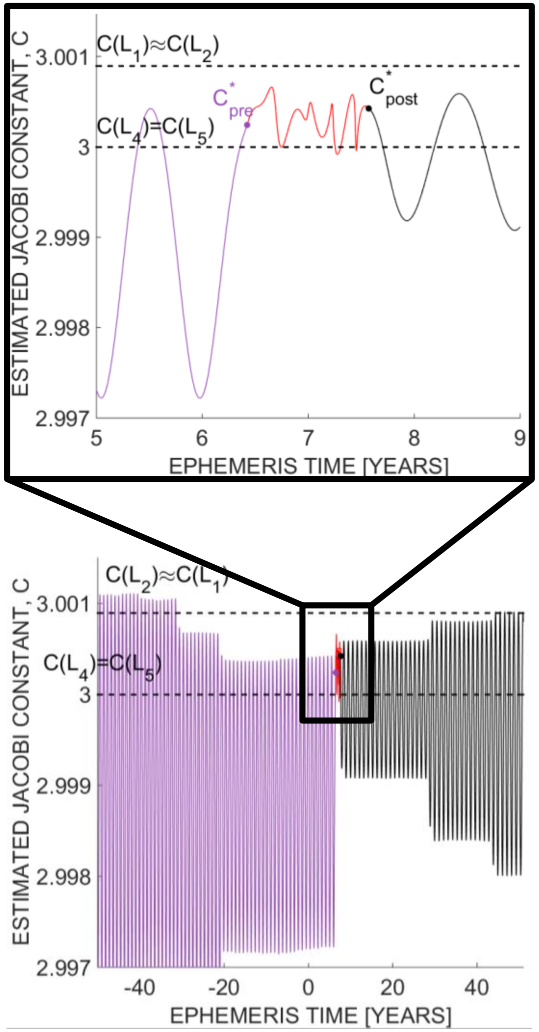
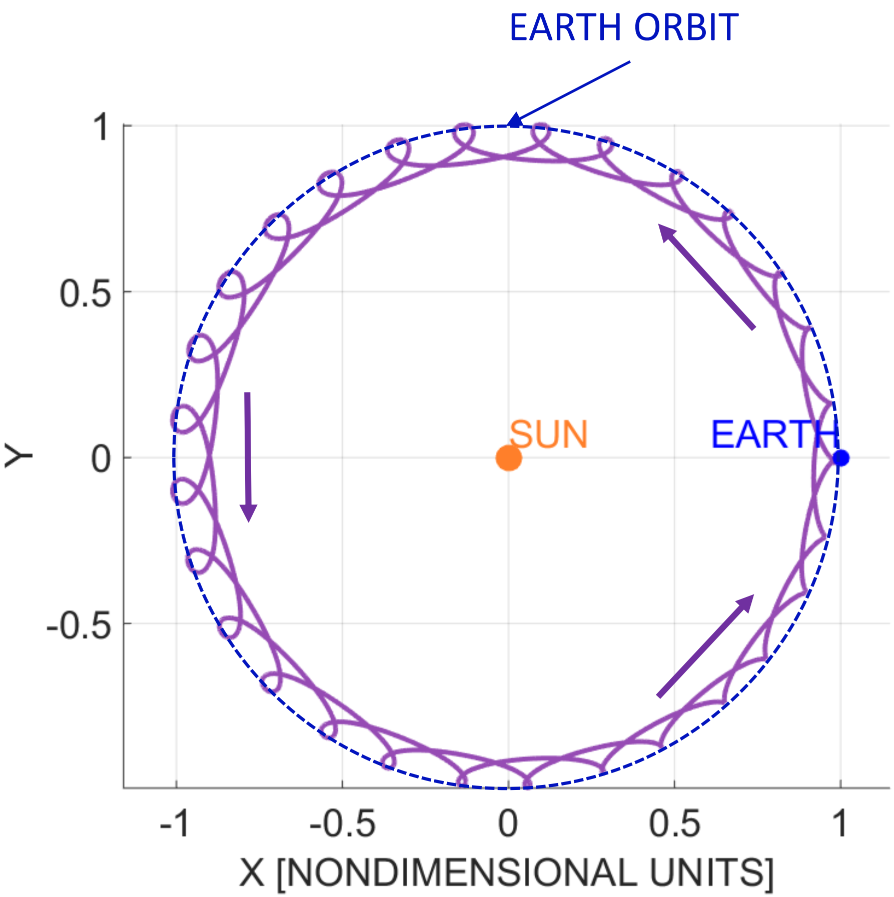
Analysis of Asteroid 2006 RH120



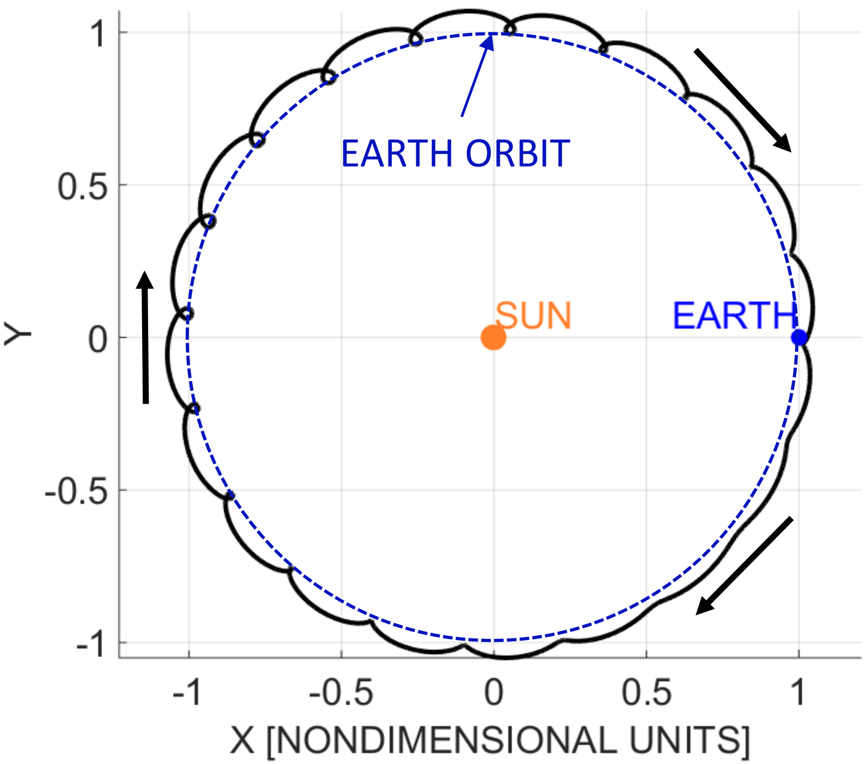
- Convert DE431 Ephemeris data to CRTBP
  - Variable Method
  - Fixed Method
- Estimate Jacobi constant for Pre- and Post-Capture Phases
  - C_{pre} = 3.000228226120707
  - C_{post} = 3.000425683288712
- Estimate Pre- and Post-Capture Resonances by 2 methods
  - CRTBP method
  - 2 body method
- Match Asteroid trajectory to invariant manifolds of periodic CRTBP orbits

Pre-Capture Resonance



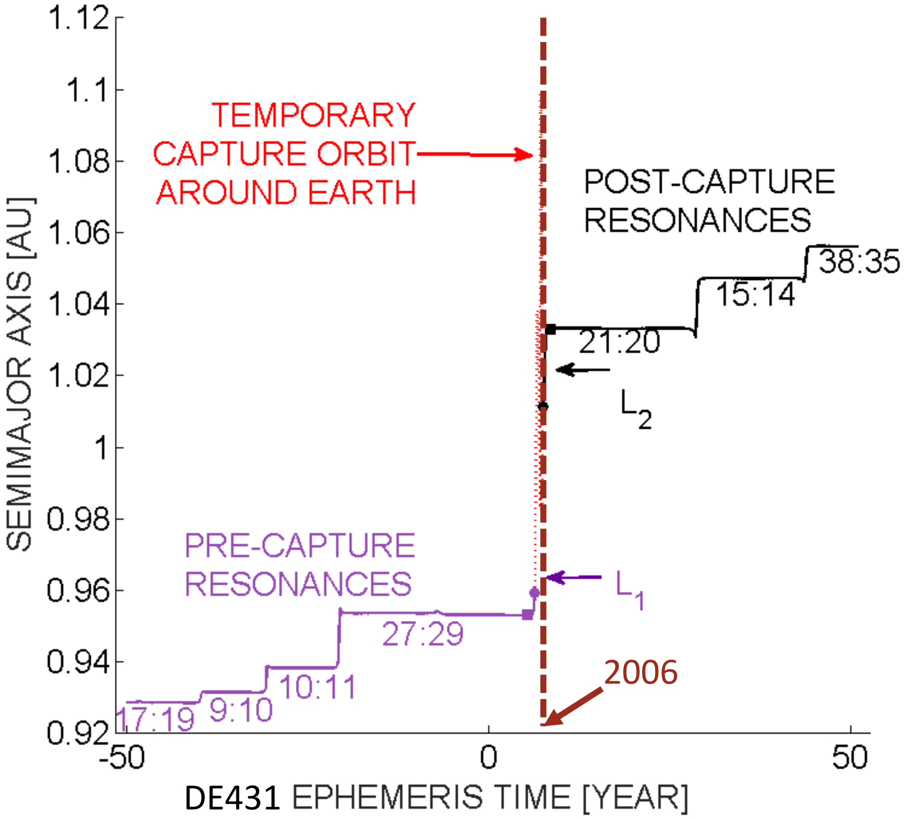
- July 1, 1979 – May 23, 2006
  - Asteroid captured 5/23/2006
  - Crossed L1 plane
- 29 heliocentric orbits
- 27 years
- **29:27 mean motion resonance**
- Previous encounter close enough to switch resonance
- 2-body period indicates 43:40 mean motion resonance
  - unlikely

Post-Capture Resonance



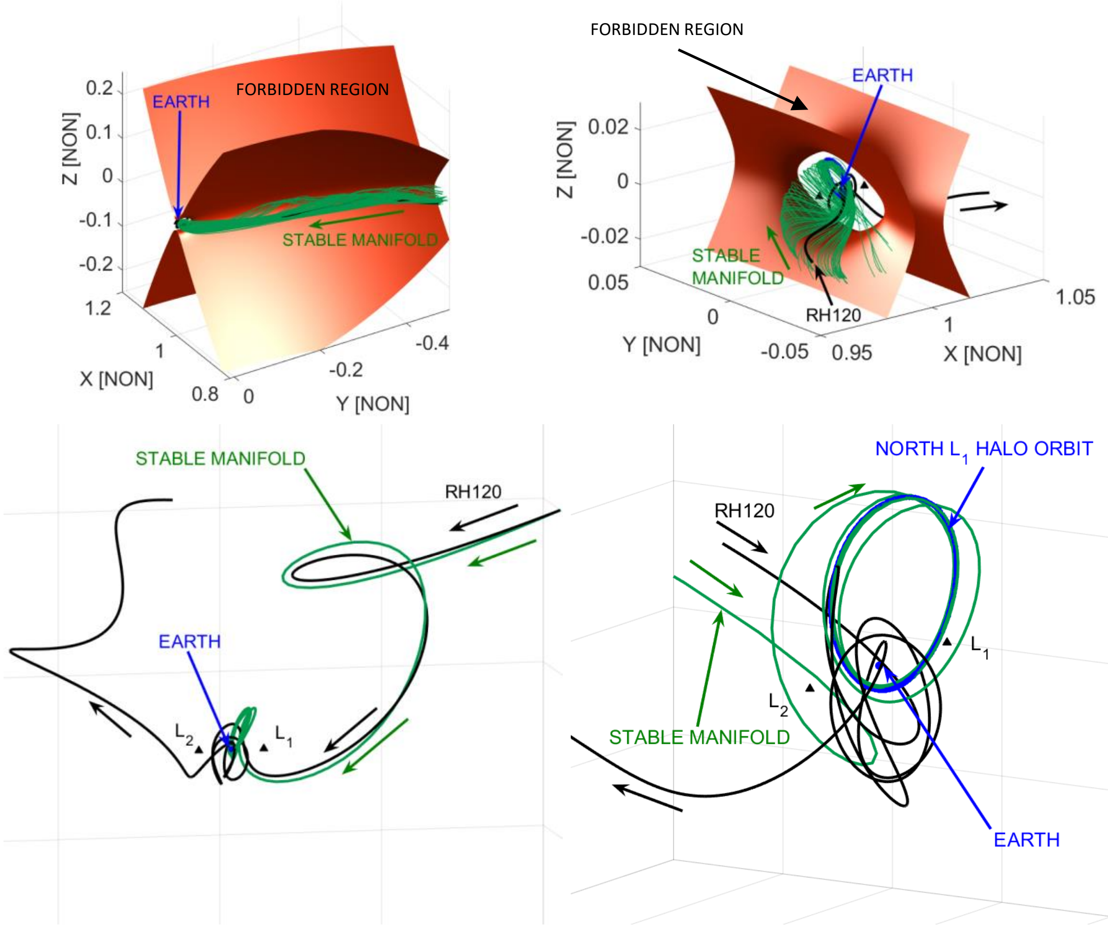
- July 28, 2007 – November 1, 2028
  - Asteroid escaped 7/28/2007
  - Crossed L2 plane
- 20 heliocentric orbits
- 21 years
- **20:21 mean motion resonance**
- Future encounter close enough to switch resonance
- 2-body period also indicates 20:21 mean motion resonance

Resonance Hopping



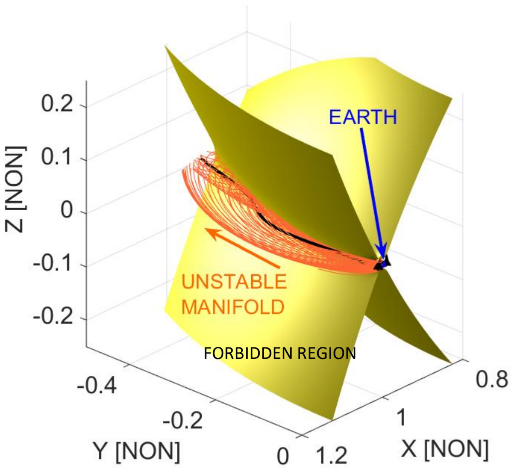
- Resonances approximated 1950-2050
- Keplerian analysis shows several resonance cycles both Pre- and Post-Capture
- Indicates several repeated near-Earth encounters
- Increasing semimajor axis

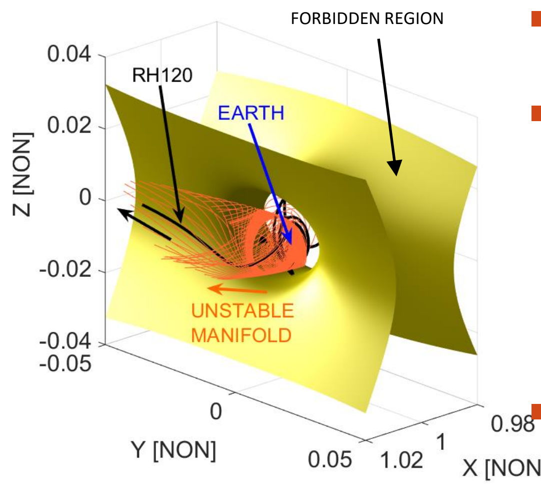
Earth Approach L_1 North Halo Orbit

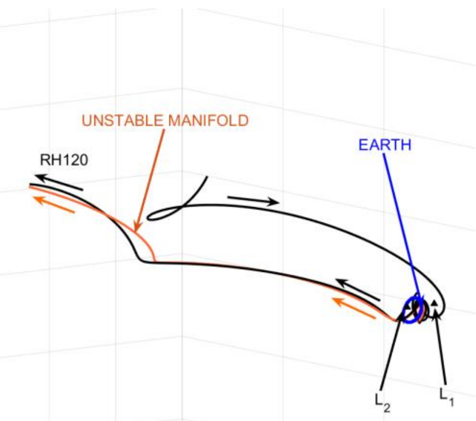


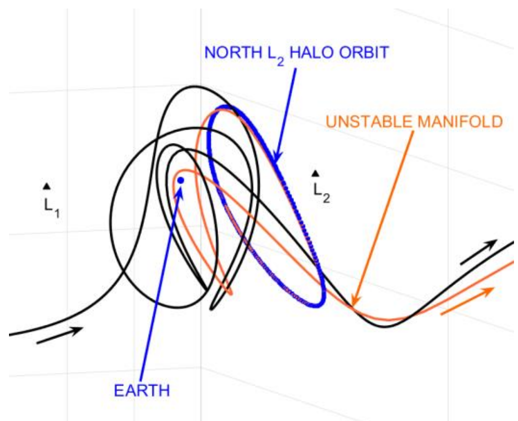
- Estimated C≈3.000228226120707
- Examined everal L_1 periodic orbits
  - Planar Lyapunov
  - Vertical Lyapunov
  - North/South Halo
- Visually compared stable manifolds to Asteroid trajectory
- “Best” match to Northern Halo orbit
- Selected single trajectory on manifold to match Asteroid trajectory

Earth Escape L_2 South Halo Orbit









- C≈3.000425683288712
- Several L_2 periodic orbits found
  - Planar Lyapunov
  - Vertical Lyapunov
  - North/South Halo
- Unstable manifolds compared visually to Asteroid trajectory
- Chosen match to Southern Halo orbit
- Single trajectory on manifold selected

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Conclusions

- Temporary Capture of Asteroid 2006 RH120 seems to be controlled by the invariant manifolds of periodic orbits in the CRTBP
  - Approach through stable manifold of L_1 North Halo Orbit
  - Escape through unstable manifold of L_2 South Halo Orbit
- Resonance cycles between repeated Earth encounters are long with mean motion resonances near 1:1
- Repeated mean motion resonance transitions near 1:1 resonance
  - This allows for trajectories with low energy levels near libration orbits
  - This enables temporary captures by Earth
- Asteroid had several near encounters in the past and is predicted to have more in the future.
  - Each encounter raises the heliocentric semimajor axis
  - Largest change occurred during Temporary Capture

Circular Restricted Three Body Problem (CRTBP)

Quasi-potential

$$\Omega = \frac{1-\mu}{r_1} + \frac{\mu}{r_2} + \frac{x^2+y^2}{2}$$

$$r_1 = (x+\mu)^2 + y^2 + z^2$$

$$r_2 = (x-1+\mu)^2 + y^2 + z^2$$

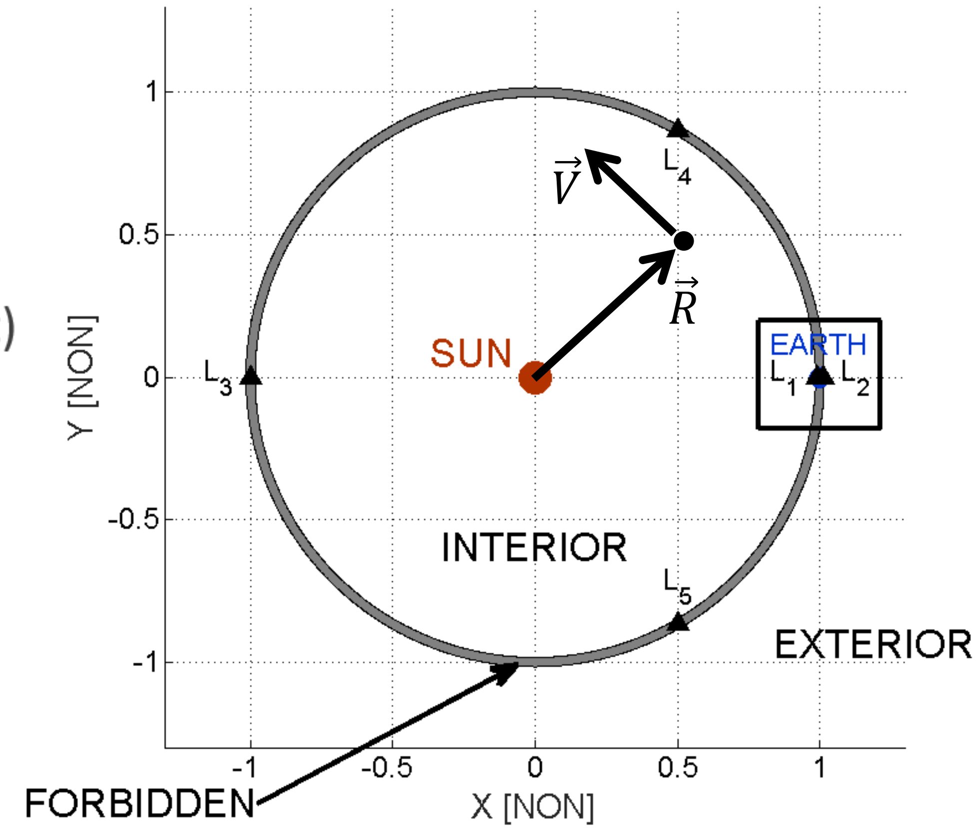
Integral of Motion (Jacobi constant)

$$C = 2\Omega - (\dot{x}^2 + \dot{y}^2 + \dot{z}^2)$$

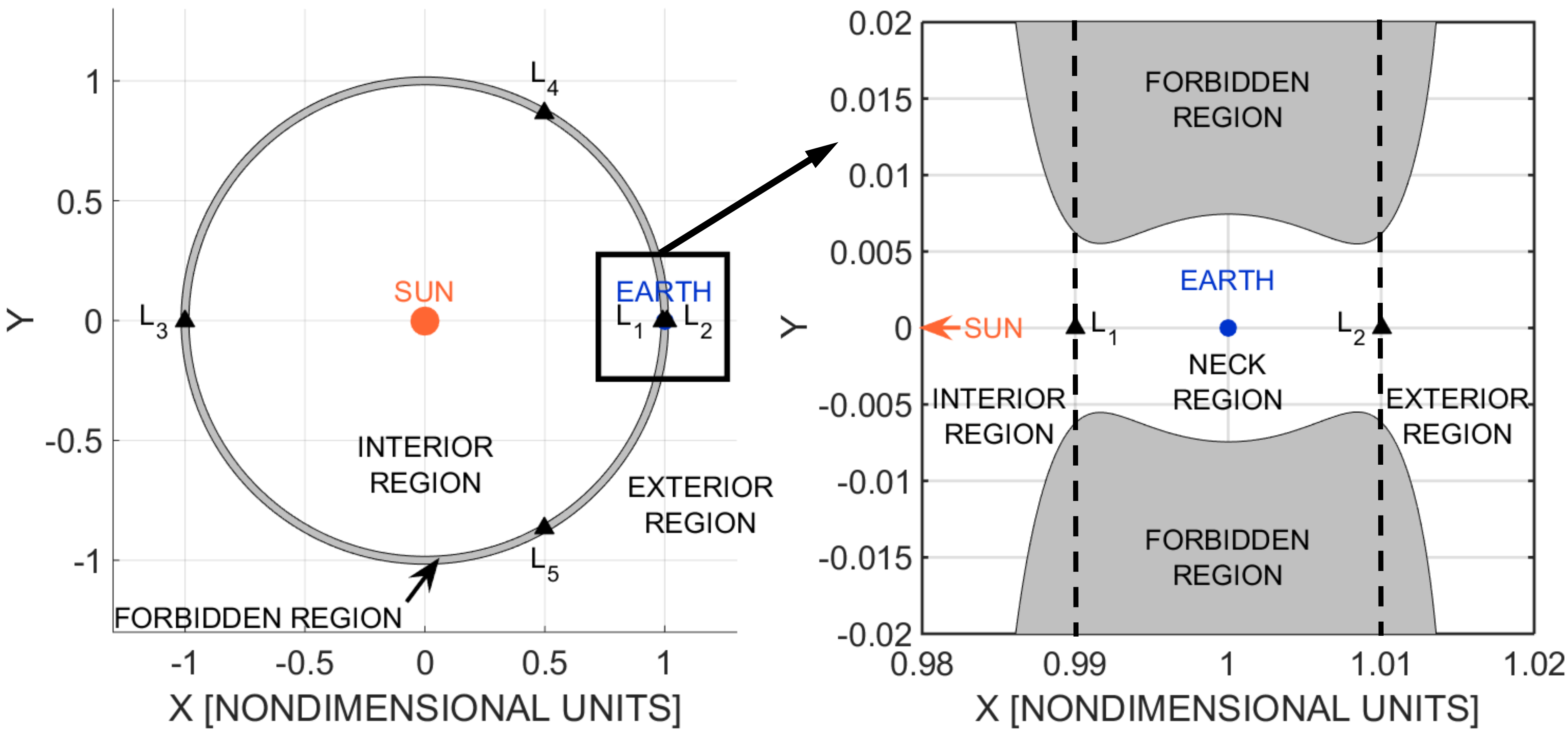
Equations of Motion

$$\vec{R} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \vec{V} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}$$

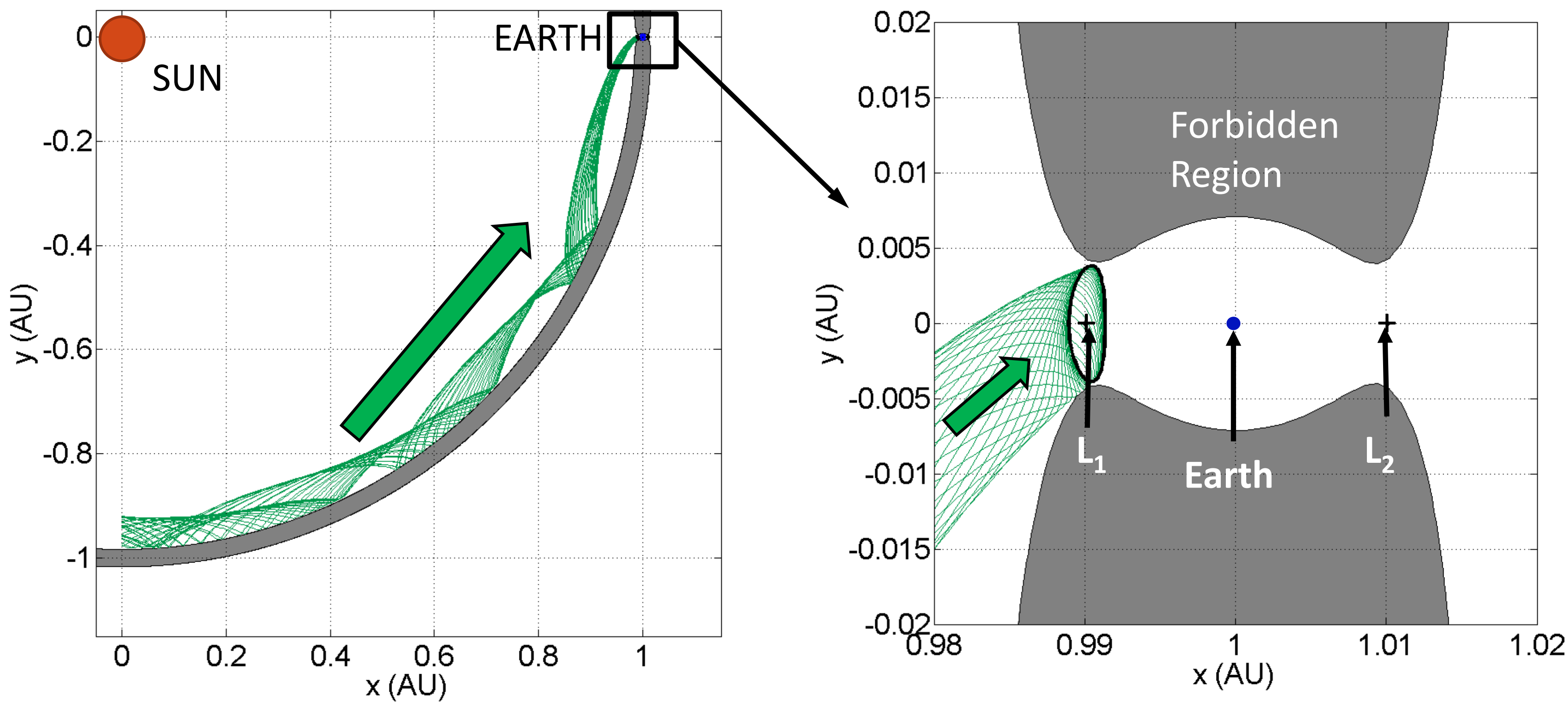
$$\frac{d^2\vec{R}}{dt^2} = \nabla\Omega$$



Circular Restricted Three Body Problem (CRTBP)



Invariant Manifolds Transit Dynamics



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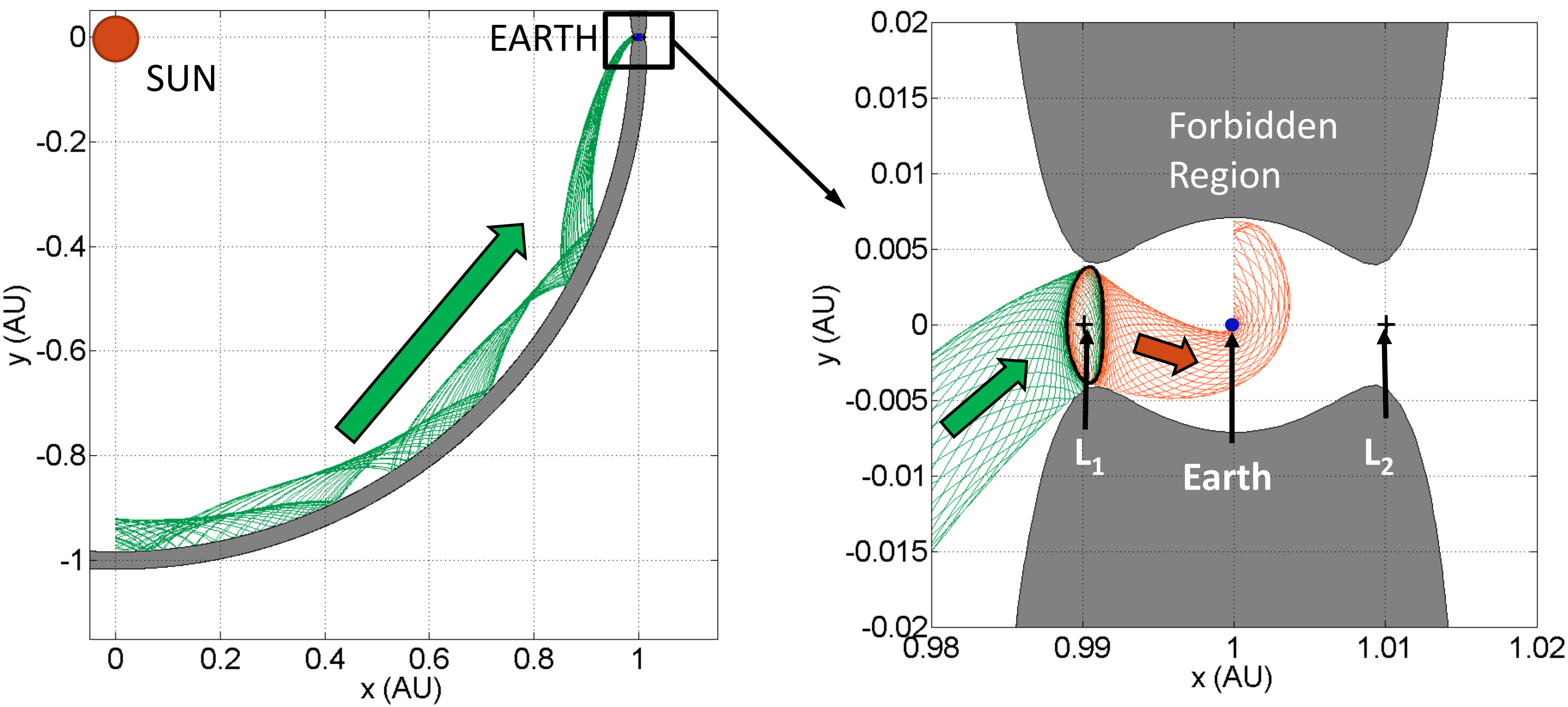
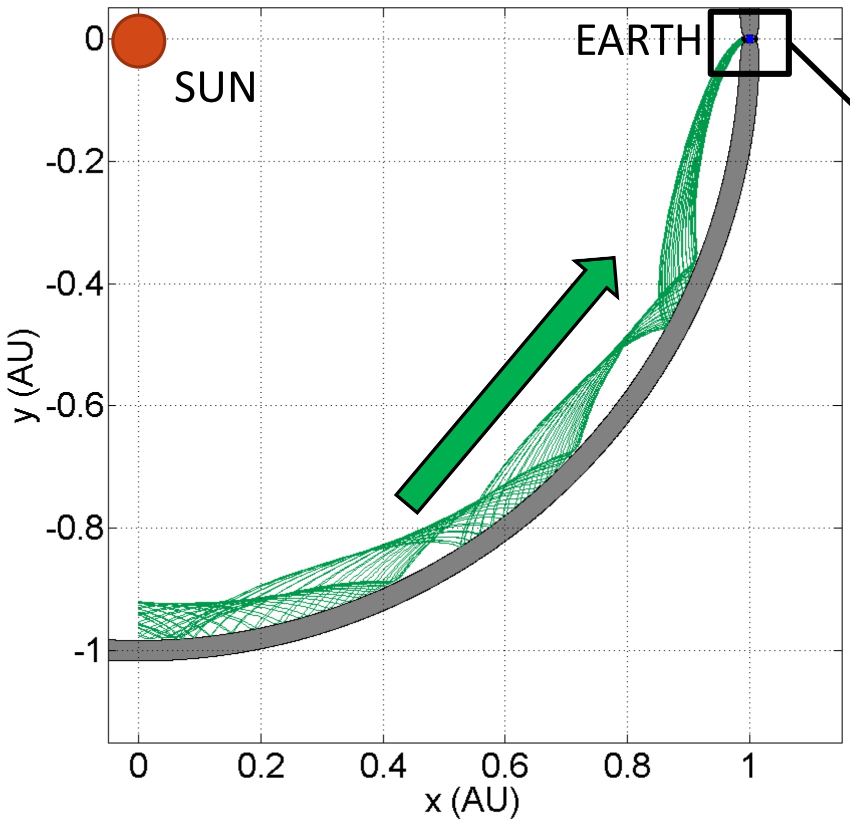
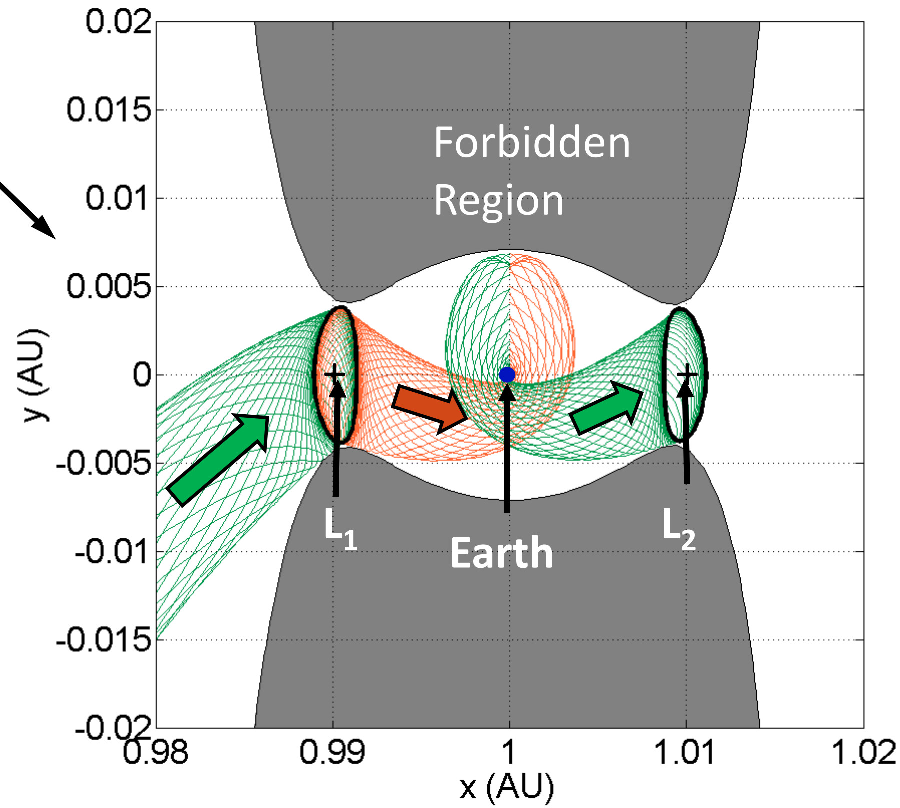
Invariant Manifolds Transit Dynamics

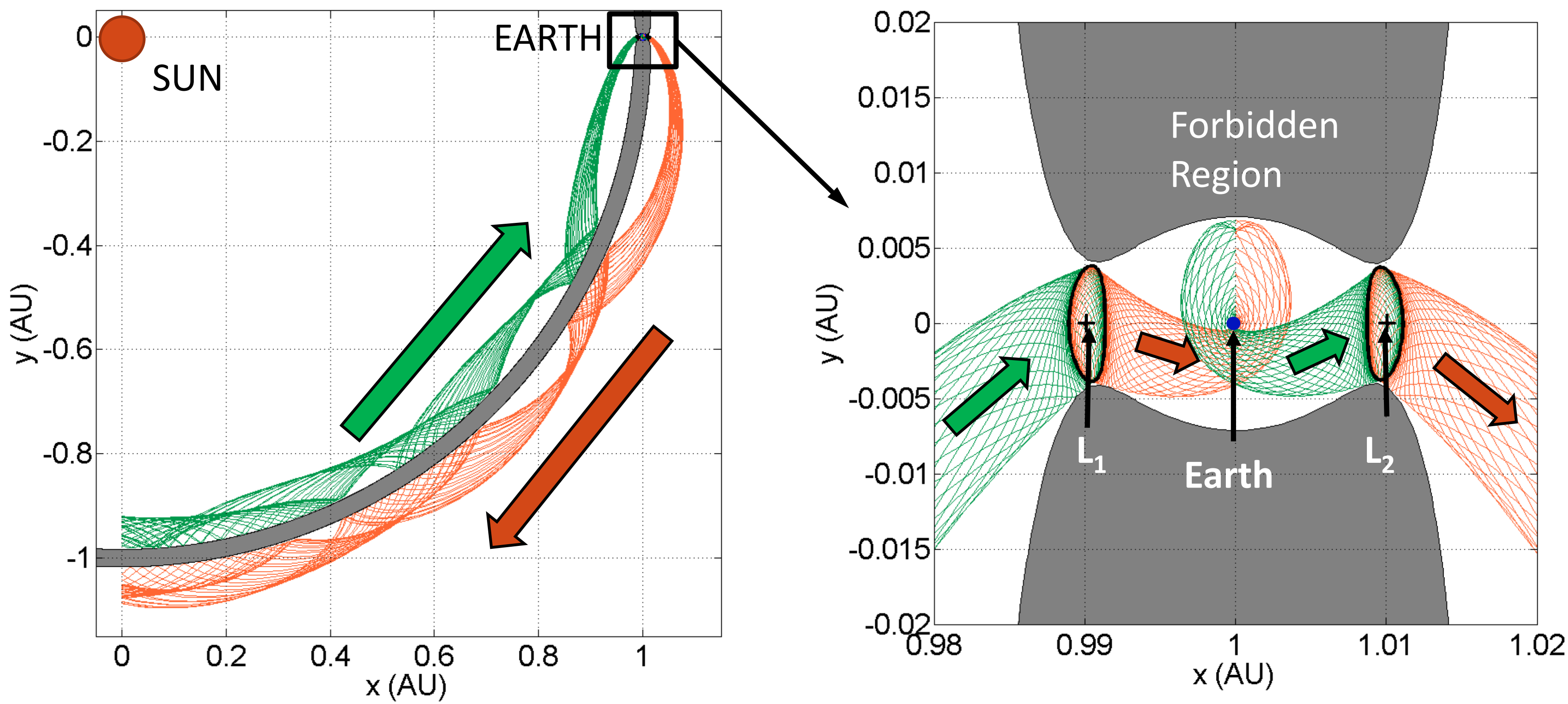


Invariant Manifolds Transit Dynamics





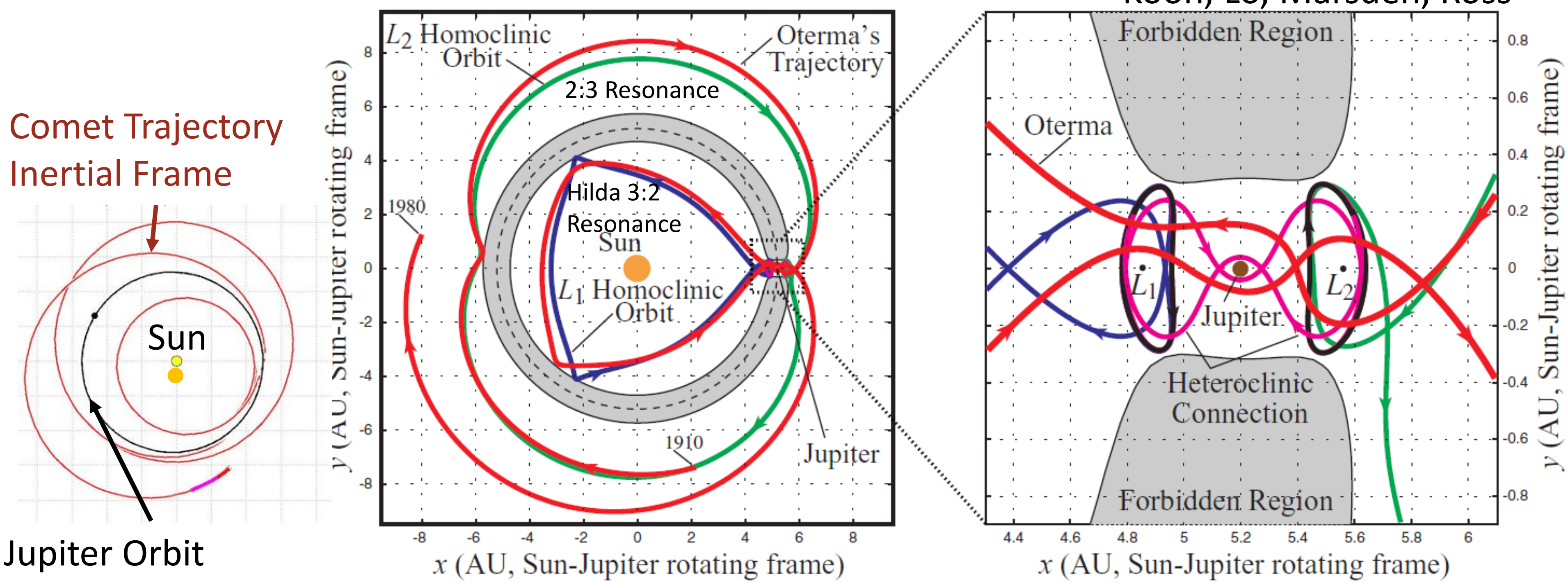
Invariant Manifolds Transit Dynamics



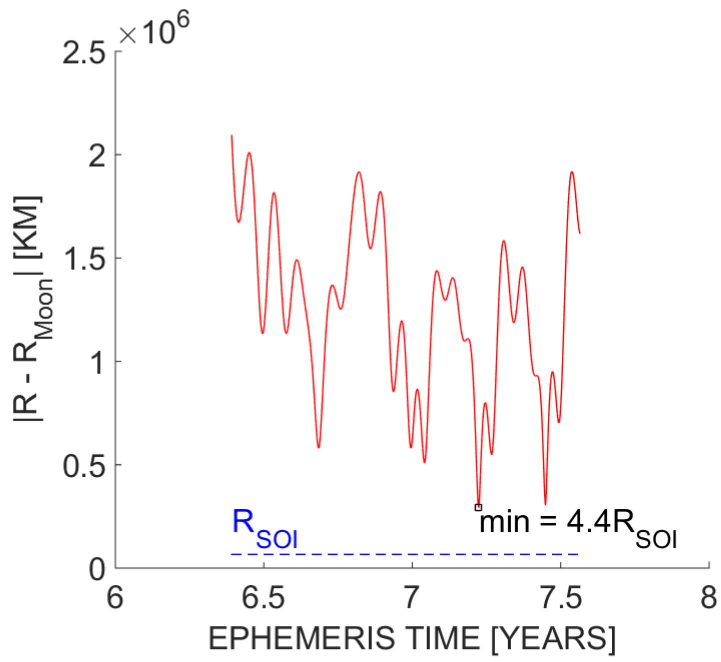
Example of Rapid Orbital Change

- Comet 39P/Oterma
- Repeated "hopping" between resonant orbits
- Heteroclinic connections between resonant orbits make this possible

Koon, Lo, Marsden, Ross



Lunar Interactions



- Lunar interactions during Temporary Capture considered
- Effects causing rapid changes not likely
- Small perturbations allowed to accumulate

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Conversion Method 1

For each t

1) R,V from ephemeris for Earth relative to Sun
2) Set length unit
3) Compute angular velocity
4) Set time and velocity units
5) Select rotating frame axes
6) Assemble Rotation matrix
7) Rotate position and velocity
8) Convert units
9) Adjust origin

1. DE431, $\vec{R}(t), \vec{V}(t)$
2. $LU = |\vec{R}|$
3. $\vec{\omega} = \frac{\vec{R}\times\vec{V}}{LU^2}$
4. $TU = |\vec{\omega}|^{-1}, VU = \frac{LU}{TU}$
5. $\hat{e}_1 = \frac{\vec{R}}{|\vec{R}|}, \hat{e}_3 = \frac{\vec{\omega}}{|\vec{\omega}|}, \hat{e}_2 = \hat{e}_3 \times \hat{e}_1$
6. $Q = \begin{bmatrix} \hat{e}_1 \\ \hat{e}_2 \\ \hat{e}_3 \end{bmatrix}$
7. $\vec{r}_D = Q\vec{r}_D^I, \vec{v}_D = Q\vec{v}_D^I - \vec{\omega} \times \vec{r}_D^I$
8. $\vec{r} = \frac{\vec{r}_D}{LU}, \vec{v} = \frac{\vec{v}_D}{VU}, t = \frac{t_D - t_{D,0}}{TU}$
9. $\vec{r} = \vec{r} + \begin{bmatrix} \mu \\ 0 \\ 0 \end{bmatrix}$

Conversion Method 2

Once

Reference R,V from ephemeris for Earth relative to Sun for unit conversion

$$\vec{R}^* = \vec{R}(t^*), \vec{V}^* = \vec{V}(t^*), LU = |\vec{R}^*|, \vec{\omega}^* = \frac{\vec{R}^* \times \vec{V}^*}{LU^2},\ TU = |\vec{\omega}^*|^{-1}, VU = \frac{LU}{TU}$$

For each t

1) R,V from ephemeris for Earth relative to Sun
2) Select rotating frame axes
3) Assemble Rotation matrix
4) Rotate position and velocity
5) Convert units
6) Adjust origin

1. DE431 $\vec{R}(t), \vec{V}(t)$
2. $\hat{e}_1 = \frac{\vec{R}}{|\vec{R}|}, \hat{e}_3 = \frac{\vec{\omega}}{|\vec{\omega}|}, \hat{e}_2 = \hat{e}_3 \times \hat{e}_1$
3. $Q = \begin{bmatrix} \hat{e}_1 \\ \hat{e}_2 \\ \hat{e}_3 \end{bmatrix}$
4. $\vec{r}_D = Q\vec{r}_D^I, \vec{v}_D = Q\vec{v}_D^I - \vec{\omega} \times \vec{r}_D^I$
5. $\vec{r} = \frac{\vec{r}_D}{LU}, \vec{v} = \frac{\vec{v}_D}{VU}, t = \frac{t_D - t_{D,0}}{TU}$
6. $\vec{r} = \vec{r} + \begin{bmatrix} \mu \\ 0 \\ 0 \end{bmatrix}$

Minimoons: Prime Targets for Rendezvous & Retrieval

- Asteroid 2006 RH120 first observed temporary moon of Earth
- Numerical studies indicate that they may be abundant
  - Granvik, Vaubaillon and Jedicke 2012: Minimoon
  - At least 1 Minimoon of diameter<1 m at any given time
  - Astronomers working to verify this NEO population
- Prime targets for potential asteroid rendezvous or retrieval
  - Minimoons have low relative speed during Temporary Capture
  - Would require less ΔV, time, cost for rendezvous or capture into long-term orbit
- We do not fully understand the dynamics involved in Temporary Capture
  - How to identify & locate potential Minimoons in NEO population?
  - What controls capture & escape of Minimoons?